A Probabilistic Detectability-Based Structural Sensor Network Design Methodology for Prognostics and Health Management

Pingfeng Wang[1], Byeng D. Youn[2,*] and Chao Hu[3]

[1]*Department of Industrial and Manufacturing Engineering, Wichita State University, Wichita, KS 67260, USA*
pingfeng.wang@wichita.edu

[2]*School of Mechanical and Aerospace Engineering, Seoul National University, Seoul, 151-742, South Korea*
bdyoun@snu.ac.kr

[3]*Department of Mechanical Engineering, University of Maryland, College Park, MD 20742, USA*
huchaost@umd.edu

ABSTRACT

Significant technological advances in sensing and communication promote the use of large sensor networks to monitor structural systems, identify damages, and quantify damage levels. Prognostics and health management (PHM) technique has been developed and applied for a variety of safety-critical engineering structures, given the critical needs of the structure health state awareness. The PHM performance highly relies on real-time sensory signals which convey the structural health relevant information. Designing an optimal structural sensor network (SN) with high detectability is thus of great importance to the PHM performance. This paper proposes a generic SN design framework using a detectability measure while accounting for uncertainties in material properties and geometric tolerances. Detectability is defined to quantify the performance of a given SN. Then, detectability analysis will be developed based on structural simulations and health state classification. Finally, the generic SN design framework can be formulated as a mixed integer nonlinear programming (MINLP) using the detectability measure and genetic algorithms (GAs) will be employed to solve the SN design optimization problem. A power transformer study will be used to demonstrate the feasibility of the proposed generic SN design methodology.[†]

[*] Corresponding author.

1. INTRODUCTION

Significant technological advances in sensing and communication promote the use of large sensor networks (SNs) to monitor structural systems, identify damages, and quantify damage levels. PHM techniques take full advantage of these advances and strive to enhance the safety and prolong the service lives of structural systems through the means of in situ data acquisition, data feature extraction and health diagnostics/prognostics to appropriately assess their health conditions and predict remaining useful lives (RULs). Through years of research efforts, PHM systems based on different types of sensors such as fiber optics, piezoelectric elements, and MEMS sensors have been developed for a wide variety of potential applications ranging from the civil, mechanical, and aerospace industries to automotive industry (Li et al., 2004; Zhao et al., 2007; Tanner et al., 2003; Ling et al., 2009; Bocca et al., 2009). Despite the worldwide attention and significant advances in maturing the technologies for practical implementation, four primary challenges still remain in PHM: (1) sensing technologies to enhance sensitivity, repeatability, robustness and reduce limited power consumptions of sensors, (2) communication techniques that allow connecting sensors with wired or wireless technology, (3) damage feature extraction research that focuses on the selection of damage features which are usually tied to methods for sensor signal processing, and (4) damage pattern recognition and prognosis methods that enable recognizing the damage state of the structure and the severity of this damage (Chang & Markmiller, 2006). As stated in the first challenge, it is clear that successful accomplishment of a structural health prognostic mission relies extremely on an effectively designed SN. Thus, one of the most important tasks in PHM system is the development of a generic SN design framework which takes into consideration different structural failure mechanisms, sensor characteristics, as well as a variety of uncertainties involved.

Most of the research activities for SN design in the past decade targeted on maximizing the coverage and minimizing the power consumption of SNs (Buczak et al., 2001; Chakrabarty & Chiu, 2002) for various applications that require the data acquisition. Research on the optimal sensor allocation has been driven by the need of optimizing large SNs for efficient monitoring activities (Chang & Markmiller, 2006; Chakrabarty & Chiu, 2001). Several methods have been developed to enhance the detection efficiency and minimize the uncertainty in decision-making based on data acquired from the SNs (Field & Grogoriu, 2006). The optimum SN was introduced as the sensor configuration that can achieve the target probability of detection. Guratzsch and Mahadevan also defined the optimum SN for structural health monitoring under uncertainties as the sensor configuration that can maximize the probability of damage detection (Guratzsch & Mahadevan, 2006). Furthermore, Li *et al* obtained a vector of sensor placement indices based on the weighted components of the mode shape matrix corresponding to the sensor position (Li et al., 2006). Ntotsios *et al* presented another approach that addresses the stochastic nature of the sensor measurements (Ntotsios et al., 2006). Azarbayejani *et al* employed an artificial neural network approach to identify the optimum sensor placement for a bridge case study (Azarbayejani et al., 2008). The sensor allocation problem is handled within the context of uncertainty and information entropy. A Bayesian method is used to quantify damage in the structure based on the change in modal information. The information entropy is used to compute a scalar measure of uncertainty in the structural damage features. A heuristic sequential sensor placement algorithm is then used to predict the optimal sensor configuration. Flynn and Todd also employed a Bayesian method for optimal sensor placement with active sensing (Flynn & Todd, 2010). Work done in the literature (Ntotsios et al., 2006; Udwadia, 1994; Heredia-Zavoni & Esteva, 1998) showed the importance of addressing the issue of uncertainty in handling the optimal sensor configuration. Other researchers (Papadimitriou et al., 2000; Kirkgaard & Brincker, 1999) also reported the use of the information entropy and information functions such as the Fisher information to formulate the objective function for optimal sensor allocations. All of the aforementioned approaches showed the significance of considering the uncertainties introduced by sensor units, structure systems as well as the operation conditions in the SN design problem and presented unique methods to deal with uncertainties in the damage detection. Most of these methods were developed for the problem of distributing a finite set of sensors to detect a specific type of structural damage and their applications are tied to and restricted by the type of failure mechanisms under consideration.

Given the significance of a SN for the PHM and years of research efforts, the design of SNs nonetheless becomes tied to the structural damage feature of choice and the development of a generic design methodology is still a hurdle to overcome. Thus, this paper presents a generic framework for the SN design optimization. First, detectability will be defined as a unified quantitative measure of SN performance in a probabilistic form; second, a detectability analysis method will be developed based on the structural simulation and health state classification; third, the SN design optimization will be formulated as an MINLP problem based on the defined detectability measure and the numerical optimizer using the genetic algorithms (GAs) will be integrated for this design optimization purpose. The developed SN design methodology will be demonstrated with a power transformer case study. The rest of the paper is organized in the following way. Section 2 will present the proposed SN design optimization framework, whereas Section 3 will present the results of the power transformer case study. A conclusion of the work will be given in Section 4.

2. DETECTABILITY-BASED DESIGN OPTIMIZATION FRAMEWORK

This section presents the detectability-based design optimization framework for structural SN design. The probabilistic detectability measure for SN performance is defined in the first subsection whereas the methodology for detectability analysis is presented in the second subsection. The SN design optimization framework for structural SN design is then presented in the third subsection.

2.1 Detectability Measure of Sensor Networks

In the proposed SN design framework, a set of health states will first be identified based on critical failure modes for the system under consideration and their combinations. The correct detection of each health state is then defined in a probabilistic form to measure the performance of a given SN design. This yields a probability of detection (PoD) matrix for a given SN design, from which the SN detectability can be derived.

Probability of Detection Matrix

The general form of a PoD matrix for a given SN design is shown in Table 1, where the P_{ij} is defined as the conditional probability that the structural system is detected to be at the HS_j by the SN given the system is at the HS_i. Clearly, P_{ij} represents the probabilistic relationship between the true health state of the system and the detected health state by the SNs. Mathematically, it is expressed as

$$P_{ij} = \Pr\,(\text{Detected as } HS_j \,|\, \text{System is at } HS_i) \qquad (1)$$

The diagonal term in the PoD matrix represents the probability of correct detection for each corresponding system health state.

Table 1: Probability of detection (PoD) matrix

Probability		Detected Health State			
		1	2	…	N_{HS}
True Health State	1	P_{11}	P_{12}	…	$P_{1N_{HS}}$
	2	P_{21}	P_{22}	…	$P_{2N_{HS}}$
	…	…	…	…	…
	N_{HS}	$P_{N_{HS}1}$	$P_{N_{HS}2}$	…	$P_{N_{HS}N_{HS}}$

Detectability

Based on the PoD matrix, the detectability for the i^{th} system health state HS_i is defined as

$$D_i = P_{ii} = \Pr\,(\text{Detected as } HS_i \,|\, \text{System is at } HS_i) \qquad (2)$$

The above detectability definition provides a probabilistic measure for the SN performance considering uncertainties involved in the SN sensing process, such as material properties for structural systems, loading conditions and operating environments. Based on this definition, the diagonal terms in the PoD matrix, which represent the probabilities of correct detection for predefined health states, will determine the overall SN performance, and thus constitute N_{HS} number of performance constraints on the detectability during the SN design optimization process. Since the detectability involves the computation of multiple conditional probabilities, an efficient and accurate method must be developed for the detectability analysis.

2.2 Detectability Analysis

Since the detectability is defined as a probabilistic measure for the performance of a SN, the detectability analysis thus needs to take into account various uncertainties involved in the structural system itself and/or the system operating condition as well. This section will present the detectability analysis method based on the structural simulation and system health state classification. The rest of this section will begin with a mathematical example of detectability calculation. Valuable information will be derived from the discussion of the example, and the detectability analysis method will then be presented.

A Mathematical Example

In this example, suppose that only one sensor will be used for the damage detection. For a healthy condition (Health State 1, HS_1), the sensor output is assumed to follow a normal distribution as N (0, 0.5^2), whereas the distribution of sensor output will be changed to N (1, 0.8^2) if there is a minor damage in the system (Health State 2, HS_2). If there is a severe damage in the system (Health State 3, HS_3), the sensor output will further increase and follow a normal distribution as N (5, 1^2). In what follows, we will find out the detectability values for all three defined health states based on the available information.

To calculate the detectability for each health state, it is necessary to classify any given sensory data into one of the three health states. This can be accomplished simply by defining the normalized distance between the sensory data and the center data point for each health state, and consequently the given set of sensor point will be classified into the health state which has the smallest normalized distance. In this example, the neutral point $X_{1\text{-}2}$ between HS_1 and HS_2 can be calculated as

$$\frac{X_{1-2} - 0}{0.5} = \frac{1 - X_{1-2}}{0.8} \qquad (3)$$

which provides $X_{1\text{-}2}$ = 0.3846. Similarly, the neutral point $X_{2\text{-}3}$ between HS_2 and HS_3 can be calculated as

$$\frac{X_{2-3} - 1}{0.8} = \frac{5 - X_{2-3}}{1} \qquad (4)$$

which provides $X_{2\text{-}3}$ = 2.7778. Figure 1 shows the sensor outputs at different health states, $X_{1\text{-}2}$, and $X_{2\text{-}3}$.

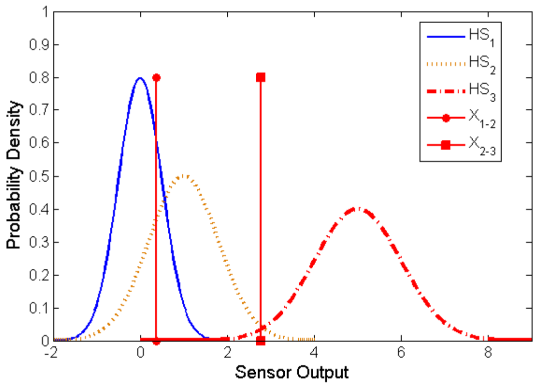

Figure 1: Sensor Outputs and Neutral Points between Health States

Based on the definition in Eq. (1), the detectability of each health state in this mathematical example can be calculated as

$$\begin{aligned} D_1 = P_{11} &= \Pr(\text{Detected as } HS_1 \mid \text{System is at } HS_1) \\ &= \Pr\left(X \le X_{1-2} \mid X \sim N\left(0, 0.5^2\right)\right) \\ &= 0.7791 \end{aligned} \tag{5}$$

$$\begin{aligned} D_2 = P_{22} &= \Pr(\text{Detected as } HS_2 \mid \text{System is at } HS_2) \\ &= \Pr\left(X_{1-2} \le X \le X_{2-3} \mid X \sim N\left(1, 0.8^2\right)\right) \\ &= 0.7660 \end{aligned} \tag{6}$$

$$\begin{aligned} D_3 = P_{33} &= \Pr(\text{Detected as } HS_3 \mid \text{System is at } HS_3) \\ &= \Pr\left(X \ge X_{2-3} \mid X \sim N\left(5, 1^2\right)\right) \\ &= 0.9869 \end{aligned} \tag{7}$$

From the analytical calculation of the detectability in the example above, it is clear that the classification of the health states and the statistical distributions of sensor outputs are crucial for the SN detectability analysis. However, in most engineering applications, an SN is always composed of multiple sensors and required to deal with much more than three different health states. Consequently, the analytical analysis of SN detectability through the calculation of neutral points between health states becomes practically impossible. Besides, the statistical distributions of all sensors' outputs for all health states are usually not available. Instead, only a finite set of sensory data might be available as training data set to characterize the sensor output for each system health state. Thus, a more sophisticated health state classifier, which should be able to classify any given set of multi-dimensional sensory data into multiple different system health states based on a finite set of training data, is needed for the SN detectability analysis. In this study, the Mahalanobis distance (MD) classifier is employed for this classification purpose.

MD Classifier for Health State Classification

The Mahalanobis distance provides a powerful method of measuring how similar one set of sensor output data is to another predefined set of training data, and can be very useful for identifying which predefined health state is the most similar one to the current system health state for the purpose of the health state classification. The MD classifier quantitatively measures the similarity between a given sensory data set and the training data sets for the i^{th} system health state through the MD, expressed as

$$MD_i = \left(X - M_i\right)^T \Sigma^{-1} \left(X - M_i\right) \tag{8}$$

where X is the given sensory data set to be classified, M_i is the vector of mean values of the training data set for HS_i, and Σ is the covariance matrix of the training data set for HS_i. The given sensory data set will be classified by the classifier into a predefined system health state that gives the smallest MD, or in other words the highest similarity. The following mathematical example demonstrates the system health state classification using the MD classifier.

In this example, two sensors are used and four system health states including one healthy state HS_1 and 3 faulty states HS_2 to HS_4 are predefined with 10 sets of data for each health state as the training data sets as shown in Table 2. To demonstrate the MD classifier, there are 5 sets of sensory data in total, as shown in the first two columns of Table 3, need to be classified into one of the four predefined health states. Using the MD classifier, the MDs for each sensory data set can be calculated with the training data set shown in Table 2 using Eq. (8). The MD values together with the classified system health state for each sensory data set are also shown in Table 3.

Based on the above procedure, the PoD matrix can be evaluated as follows. Suppose that there are totally T_i number of testing sensory data sets for HS_i, and in which after the classification process T_{ij} sets classified into HS_j , where i, j = 1, 2, …, N_{HS}, then based on the definition of the PoD matrix, the probability of detection P_{ij} can be approximately calculated as

$$P_{ij} \simeq \frac{T_{ij}}{T_i} \tag{9}$$

Besides, since one set of sensory signal will definitely be classified into one of the predefined N_{HS} health states, thus,

$$\sum_{j=1}^{N_{HS}} T_{ij} = T_i \tag{10}$$

Eq. (10) Indicates that the summation of each row of the PoD matrix equals to one. Similarly, the detectability for HS_i can be obtained as

$$D_i = P_{ii} \simeq \frac{T_{ii}}{T_i} \tag{11}$$

Table 2: Characteristic Data for System Health States

HS_1		HS_2		HS_3		HS_4	
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
-0.22	-0.09	1.15	-1.20	1.20	-2.51	-1.00	1.28
-0.83	0.36	0.33	-0.66	2.13	-1.69	-1.16	0.87
0.06	-0.29	1.36	-0.59	1.47	-1.75	-0.45	0.81
0.14	1.09	1.81	-0.64	2.71	-1.15	-1.94	0.85
-0.57	-0.07	0.65	-0.35	1.60	-1.70	-0.79	0.26
0.60	0.06	1.43	-0.67	2.26	-2.32	-0.55	0.88
0.59	0.53	1.63	-0.40	2.11	-1.81	-0.63	1.06
-0.02	0.03	0.20	-1.60	1.54	-2.50	-0.71	1.16
0.16	-0.05	0.28	-1.01	0.91	-2.01	-0.98	1.72
0.09	-0.42	1.29	-1.08	1.97	-2.02	-0.66	0.82

Table 3: System Health States Classification Using MD Classifier

Sensory Data		Mahalanobis Distance				Classified State
S_1	S_2	HS_1	HS_2	HS_3	HS_4	
0.74	-1.05	10.75	0.39	17.68	40.42	HS_2
1.59	-2.12	42.55	18.16	0.20	94.23	HS_3
-0.93	1.22	11.81	64.75	169.94	0.43	HS_4
-0.12	-0.21	0.58	10.63	62.58	12.28	HS_1
1.89	-1.97	44.81	17.91	0.07	96.03	HS_3

Procedure of Detectability Analysis

Based on the preceding discussion, defining the system health states is crucial for the SN design, which will determine the functionality of the SN to be designed. Through defining different type of system health states, SNs can be designed to tackle different failure mechanisms for structural systems. After defining the health states, collecting sample training and testing data sets for each health states are the next step, which can be accomplished through the structural simulation using valid numerical models, such as finite element analysis (FEA). The sample size of the training and testing data sets will determine the accuracy of the detectability evaluation using the proposed MD classifier. With the training and testing data sets available, the detectability for each predefined health state for a given SN design can be evaluated in the same way as we did in the previous example.

The overall procedure of the detectability analysis can be summarized in Table 4.

Table 4: Procedure for Detectability Analysis

STEP 1:	Define the problem and system health states;
STEP 2:	Collect characteristic training and testing data sets for each predefined system health state;
STEP 3:	Extract a corresponding subset of training and testing data, for a given SN design, from the characteristic data sets obtained in STEP 2;
STEP 4:	Perform classification using the MD classifier defined by Eq. (8);
STEP 5:	Calculate the detectability for each health states using Eq. (11)

2.3 Sensor Network Design Optimization

The appropriate selection of the sensing devices, such as fiber optics, piezoelectric, MEMS sensors, accelerometers, or acoustic sensors, is determined by the sensor's characteristic attributes, such as full-scale dynamic range, sensitivity, noise floor, and analog-to-digital converter resolution. Thus, the design variables involved in the proposed design framework are the decision variables for the selection of sensing devices, numbers of selected sensing devices, sensing device locations, and the parameters for controlling the sensing process, such as excitation frequency, loading levels. The design constraints are SN probabilistic performance requirements considering various uncertainties presented in the structures as well as the operating conditions. The performance requirements include the SN detectability for each predefined system health state. With all factors considered above, the SN design optimization problem can be formulated as:

$$\begin{aligned} &\text{Minimize} \quad C \\ &\text{subject to} \quad D_i(\mathbf{X_T},\ \mathrm{X_N},\ \mathbf{X_{Loc}},\ \mathbf{X_s}) \geq D_i^t \\ &\qquad\qquad (i = 1, 2, \ldots, N_{HS}) \end{aligned} \tag{12}$$

where X_T is a vector of the binary decision variables for the selection of the types of sensing devices, X_N is a vector consisting of numbers of each selected type of sensing devices, X_{Loc} is a 3-D vector of the location of each sensing device, and $\mathbf{X_s}$ is a vector of sensing control parameters; N_{HS} is the total number of predefined health states for the structural system. D_i is the detectability of the SN for the i^{th} predefined health state, which is a function of the design variables X_T, X_N, X_{Loc} and $\mathbf{X_s}$, whereas D_i^t is the target SN detectability for the i^{th} predefined health state.

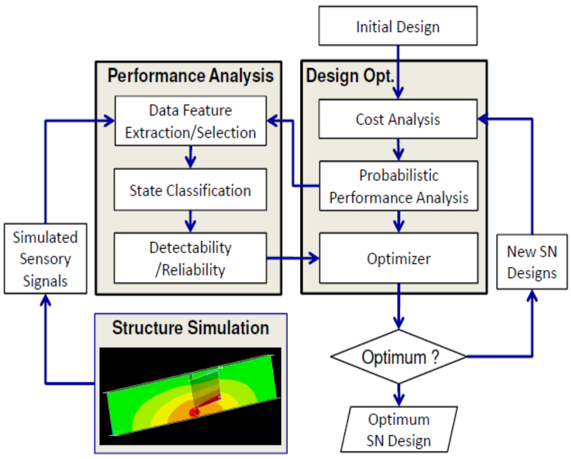

Figure 2: Flowchart of Detectability-Based Design Optimization Framework for Structural SN Design

The SN design optimization problem in Eq. (12) contains discrete decision variables for the selection of sensing devices, integer variables for the number of selected sensing devices, as well as continuous variables for the sensor locations. Thus, it is formulated as a mixed-integer nonlinear programming (MINLP) problem (Adjiman et al., 2000), and heuristic algorithms such as Genetic Algorithms (GAs) can be used as the optimizer to for the optimization purpose. In this study, the GA is employed for the example problem that will be detailed in the subsequent section. More alternative algorithms for solving the MINLP problem can be found in references (Adjiman et al., 2000; Wei & Realff, 2004).

Figure 2 shows the flowchart of the SN design optimization process. As shown in the figure, the process starts from an initial SN design and goes into the design optimization subroutine (the right hand side grey box), which will carry out the SN cost analysis, call the performance analysis subroutine (the left hand side grey box) to evaluate the performance of the SN at the current design, and execute the optimizer to generate the new SN design if the optimality condition is not met. In the performance analysis subroutine, the detectability analysis as discussed in the previous section will be carried out. Before solving the optimization problem, valid system simulation models have to be built and structural simulations have to be accomplished so that the training and testing data sets for each predefined health state are available.

3. SENSOR NETWORK DESIGN AGAINST POWER TRANSFORMER MECHANICAL FAILURE

Power transformers are among the most expensive elements of high-voltage power systems. The monitoring of power transformers enables the transition from the traditional time-based maintenance to the condition-based maintenance, resulting in significant reductions in maintenance costs (Leibfield, 1998). Due to the difficulties of direct measurement inside the transformer, the data that are actually most often used for both diagnosis and prognosis of transformers are obtained through indirect measurements (Rivera et al., 2000). For example, measurements of temperature are firstly accomplished at accessible points and a modeling of the gradient can then be used to induce the maximum temperature in some areas; electric parameters and analysis of moisture content of the cooling oil are often performed for the diagnosis and condition-based maintenance of transformers, with frequency response analysis of electric characteristics being common (Allan et al., 1992); the vibrations of the magnetic core and of the windings could characterize transitory overloads and permanent failures before any irreparable damage occurs. This case study aims at designing an optimum SN on the front wall surface of a power transformer. The measurements of the transformer vibration responses induced by the magnetic field loading enables the detection of mechanical failures of winding support joints inside the transformer.

3.1 Description of the Case Study

In this study, the winding support joint loosening is considered as the failure mode, the detection of which will be realized by collecting the vibration signal, induced by the magnetic field loading with a fixed frequency on the power transformer core, using the optimally designed SN at the external surface of the transformer. The validated finite element (FE) model of a power transformer was created in ANSYS 10 as shown in Figure 3, where one exterior wall is concealed to make the interior structure visible. Figure 4 shows 12 simplified winding support joints with 4 for each winding. The transformer is fixed at the bottom surface and a vibration load with the frequency of 120 Hz is applied to the transformer core. The joint loosening was realized by reducing the stiffness of the joint itself. Different combinations of the loosening joints will be treated as different health states of the power transformer which will be detailed in the next subsection.

The uncertainties in this case study are modeled as random parameters with corresponding statistical distributions listed in Table 5, which includes the material properties, such as Young's modulus's, densities, Poisson ratios, for support joints and windings, as well other parts in the power transformer system. Besides, the geometry parameters are also considered as random variables. These uncertainties will be propagated into the structural vibration responses and will be accounted for when designing an optimum SN.

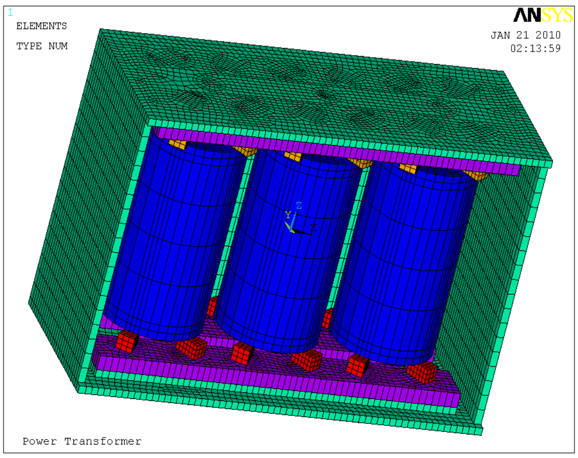

Figure 3: A Power Transformer FE Model (without the covering wall)

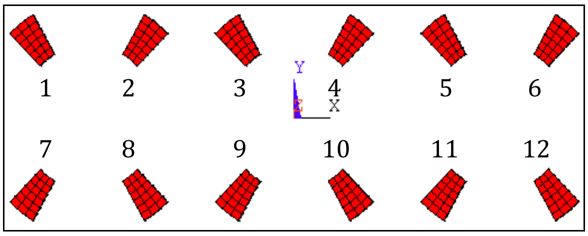

Figure 4: Winding Support Joints and Their Numberings

Table 5: Random Property of the Power Transformer

Random Variable	Physical Meaning	Randomness (cm, g, degree)
X_1	Wall Thickness	N (3, 0.06^2)
X_2	Angular width of support joints	N(15, 0.3^2)
X_3	Height of support joints	N(6, 0.12^2)
X_4	Young's modulus of support joint	N(2e12, $4e10^2$)
X_5	Young's modulus of loosening joints	N(2e10, $4e8^2$)
X_6	Young's modulus of winding	N(1.28e12,$3e10^2$)
X_7	Poisson ratio of joints	N(0.27, 0.0054^2)
X_8	Poisson ratio of winding	N(0.34, 0.0068^2)
X_9	Density of joints	N(7.85, 0.157^2)
X_{10}	Density of windings	N(8.96, 0.179^2)

3.2 Health States and Simulations

For the purpose of demonstrating the proposed SN design methodology, 9 representative health states (see Table 6) were selected from all possible combinations of 12 winding support joint failures. Among these 9 selected health states, HS_1 denotes the healthy condition without any loosening joint, whereas HS_2 to HS_9 are health states with either one or two loosening joints. According to the statistical properties of random parameters in Table 5, 200 sets of random samples were generated and the simulations for each of 9 health states were carried out and the vibration response of the displacement amplitudes for all the finite element nodes on the outer wall surfaces were saved as the simulation results. The stress contour of the healthy state power transformer at the nominal values of the random parameters from the structural simulation is shown in Figure 5, whereas the vibration response of the covering wall is shown in Figure 6. The first 100 sets of simulation results were used as the training data set and the others were used as testing data set. These simulation results were later used to evaluate the SN detectability. As mentioned in the previous section, this case study problem is formulated as designing an SN on the surface of the covering wall of the power transformer to minimize the cost of the SN while satisfying the detectability constraints for each health state, i.e., the detectability should be greater than a target detectability of 0.95.

Table 6: Definition of System Health States

Health State	1	2	3	4	5	6	7	8	9
Loosening Joints	-	1	2	3	1,2	1,3	1,5	1,9	1,11

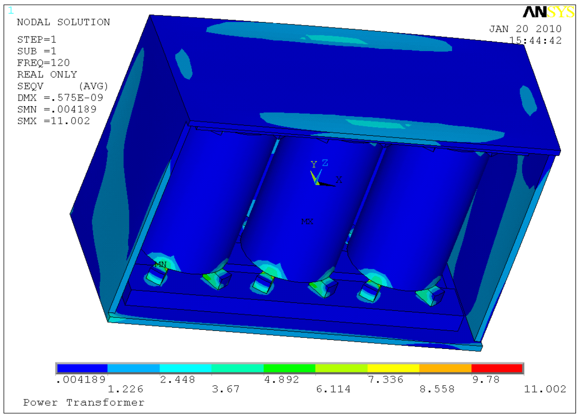

Figure 5: Stress Contour of the Winding Support for the Healthy State of Power Transformer

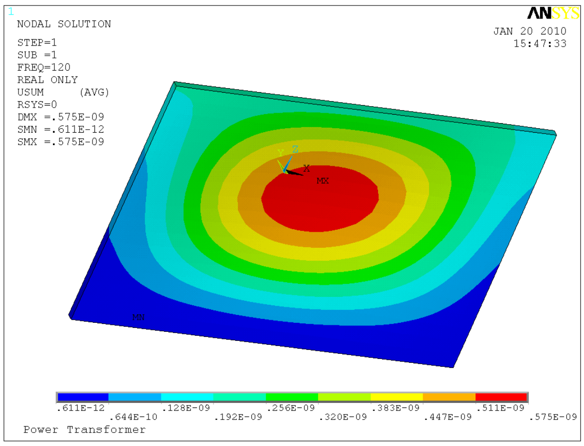

Figure 6: Vibration Displacement Contour of the Power Transformer Covering Wall for the Healthy State of Power Transformer

As the the vibration displacement amplitude of each node on the surface of the covering wall was used as the simulated sensor (accelerometer) output. Thus, the design variables in this case study include: (1) total number of accelerometers, (2) location of each accelerometer, and (3) the direction (X or Z) of each accelerometer.

3.3 Results and Discussion

Following the flowchart shown in Figure 2 and the detectability analysis procedure listed in Table 4, the SN design problem in this case study was solved using the genetic algorithm. Figure 7 shows the detectability for each of 9 health states at the optimum SN design versus different total numbers of sensors. With the target detectability being 0.95, we obtained the optimum SN design on the outer wall surface (140cm x 90cm) with totally 9 sensors, as shown in Table 7 and Figure 8. The detectability values of all 9 health states under the optimum SN design are summarized in Table 8.

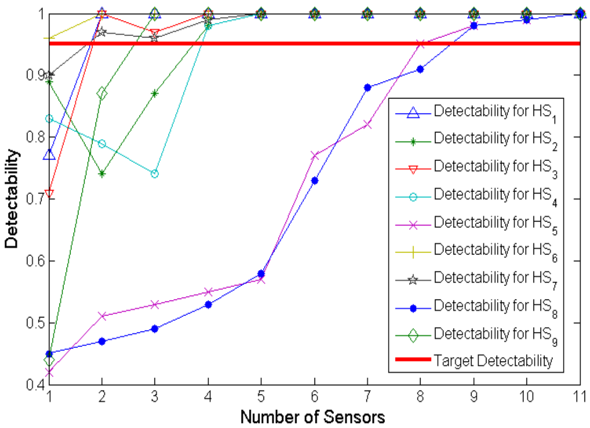

Figure 7: Minimum Detectability at Optimum Design versus Total Number of Sensors

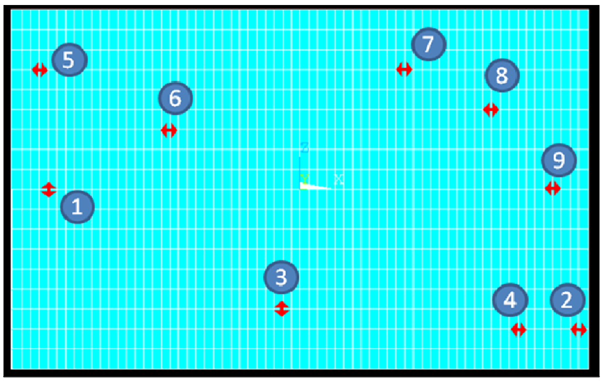

Figure 8: Optimal Design of the Distributed SN for Power Transformer Case

Table 7: Optimum SN Design for Power Transform Case Study

Sensor Index	Location (cm)		Direction
	x	z	
1	-56.4	0.0	Z
2	67.2	-34.4	X
3	-2.6	-30.0	Z
4	49.7	-34.4	X
5	-57.9	30.0	X
6	-30.6	15.3	X
7	27.5	30.0	X
8	39.3	35.2	X
9	59.1	0.0	X

Table 8: Detectability under Optimum SN Design for Power Transform Case Study

HS_1	HS_2	HS_3	HS_4	HS_5	HS_6	HS_7	HS_8	HS_9
1	1	1	1	0.98	1	1	0.98	1

The results of the power transformer case study demonstrate that the proposed SN design framework is capable to tackle the SN design problems for complicated engineering systems with multiple system health states and a variety of system input uncertainties. The authors also would like to address the following comments for the readers to better understand the problem. Firstly, in this case study, the GA was implemented for the design optimization and repeatedly executed for 10,000 times. Although, for most of times, the optimization converged to the optimal design, the convergence to local minima was also observed. Thus, it would be interesting to investigate other optimization algorithms (e.g., the particle swarm optimization) (Valle et al., 2008) to make the SN design process more robust; secondly, due the computational time, only 100 samples were simulated for each health state, resulting in 2 decimal digits of precision in the detectability estimates. To obtain more accurate results, more samples from the structural simulation are needed. Lastly, to make the SN design more reliable, the redundancy could be easily integrated to the proposed SN design framework by adding the redundancy as an additional set of design variables and the SN reliability as an additional constraint.

4. CONCLUSION

This paper presented the generic design framework for SN design optimization using the detectability measure while accounting for uncertainties in material properties and geometric tolerances. The proposed work defined the detectability measure to quantify the performance of a designed SN in a probabilistic form. Then, detectability analysis was developed based on structural simulation and health state classification, where the Mahalanobis distance classifier was proposed for health state classification. Finally, the generic SN design framework was formulated as a mixed integer nonlinear programming (MINLP). The genetic algorithm was used as the optimizer to solve the SN design optimization problem. The power transformer case study demonstrated that the proposed generic SN design framework is feasible to handle multiple failure modes and uncertainties in material properties and geometric tolerances.

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NOMENCLATURE

HS_i	=	The i^{th} health state of the system
P_{ij}	=	Probability of detection (PoD)
D_i	=	Detectability of the i^{th} health state
MD	=	Mahalanobis distance
Pr (.|.)	=	Conditional probability
M_i	=	Vector of mean values
Σ	=	Covariance matrix
T	=	Matrix inverse
C	=	Cost function

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Pingfeng Wang: Dr. Wang received his B.E. degree in Mechanical Engineering from The University of Science and Technology (Beijing, China) in 2001, the M.S. degree in Applied Mathematics in Tsinghua University (Beijing, China) in 2006, and the Ph.D. degree in mechanical engineering at the University of Maryland, College Park (Maryland, USA) in 2010. He is currently an Assistant Professor in the Department of Industrial and Manufacturing Engineering at Wichita State University. His research interests are system reliability and maintainability analysis, risk-based design, health monitoring, prognostics and health management (PHM) and structural sensor network design.

Byeng D. Youn: Dr. Byeng D. Youn is currently an Assistant Professor in the School of Mechanical and Aerospace Engineering at Seoul National University in South Korea. Dr. Youn is dedicated to well-balanced experimental and simulation studies of system analysis and design and is currently exploring three research avenues: (1) system risk-based design, (2) prognostics and health management (PHM), and (3) energy harvester design. Dr. Youn's research and educational portfolio includes: (i) *six notable awards***,** including the ISSMO/Springer Prize for the Best Young Scientist in 2005 from the International Society of Structural and Multidisciplinary Optimization (ISSMO), (ii) *over one hundred publications* in the area of system risk assessment and design and PHM. His primary applications include Li-ion batteries, consumer electronics, and large-scale engineered systems (e.g., automotives).

Chao Hu: Mr. Hu received his B.E. degree in Engineering Physics from Tsinghua University (Beijing, China) in 2003. He is currently pursuing the Ph.D. degree in mechanical engineering at The University of Maryland, College Park (Maryland, USA). His research interests are system reliability analysis, prognostics and health management (PHM), and battery power and health management of Li-ion battery system.