

Stochastic Characterization and Update of Fatigue Loading for Mechanical Damage Prognosis

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ABSTRACT

Accurate characterization and prediction of loading, while properly accounting for uncertainty, are essential for probabilistic fatigue damage prognosis. Three different techniques, including rainflow counting, the Markov chain method, and autoregressive moving average (ARMA) method, are reviewed for stochastic characterization and reconstruction of the fatigue load spectrum for prognosis. The ARMA method is extended by introducing random coefficients and probabilistic weights, to account for the uncertainty in the selection of models, inherent variability of loading, and uncertainty due to sparse data. A continuous model updating framework based on usage monitoring data is developed and applied to all the three techniques mentioned above. The relation between prediction accuracy and updating period is evaluated quantitatively. A quantitative model validation metric is proposed for assessing the accuracy of load prediction.*

1 INTRODUCTION

The fatigue process of mechanical components under service loading is stochastic in nature. Therefore, sampling based methodologies for uncertainty quantification and propagation in fatigue analysis have been developed (Doebeling & Hemez, 2001; Farrar & Lieven, 2007; Gupta & Ray, 2007; Pierce, Worden, &

Bezazi, 2008; Sankararaman, Ling, Shantz, & Mahadevan, 2009), to address probabilistic crack growth and life prediction. Three types of uncertainty sources, including natural variability, data uncertainty, and model uncertainty, have been considered. Among the various sources of uncertainty, past experimental studies have suggested that the variability and the uncertainty from load spectrum have significant influence on crack growth behavior and fatigue life (Moreno, Zapatero, & Dominguez, 2003; Zapatero, Moreno, Gonzalezherrera, & Dominguez, 2005; Wei, Delosrios, & James, 2002). In addition to the extensive efforts that have been devoted to generate deterministic load spectra (Heuler & Klatschke, 2005; Xiong & Sheno, 2008), it is desirable to characterize the uncertainty in the load spectrum based on usage monitoring data, and provide future load prediction for damage prognosis.

Two types of methods have been developed to model the fatigue load spectrum, namely cycle counting methods and random process methods. The cycle counting methods employ counting algorithms on load amplitude data based on certain definitions of cycles, and then extract counting matrices containing the information on the number of cycles, the mean value and the range of each cycle (ASTM, 2005). Among various cycle counting methods, the rainflow counting method has been considered as the most efficient and accurate (Dowling, 1972). Further, following certain rules, load history can be regenerated from the counting matrices (Khosrovaneh & Dowling, 1990). The cycle counting methods are simple to implement and can be directly used to estimate fatigue damage due to the applied loading, by using an S-N curve-based fatigue damage cumulative law, such as the Palmgren-Miner linear rule (Miner, 1945). It should be noted that the counting matrices contain no information on the correlation between load cycles and hence the corresponding reconstruction is simply a

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procedure of randomly rearranging cycles with exactly the same numbers as the original load spectrum data.

The random process methods characterize fatigue load spectrum as a stochastic process. The Markov chain method treats loading as a discrete time Markov chain with stationary transition probability (Krenk & Gluwer, 1989; Rychlik, 1996). The advantage of the Markov chain method is that it retains the correlation between adjacent turning points (load extrema), and simulations of load spectrum are fast once the transition probability matrix has been established. Note that load amplitudes are discretized into different levels in the Markov chain method, as well as the cycle counting method, and hence a relatively large transition matrix is required if the variation of loading amplitude is high. Further, it is assumed that the next turning point depends only on the previous turning point. This assumption may not be valid if strong autocorrelations exist in the load spectrum.

Frequency domain-based methods and time domain-based methods have been investigated to model the load spectrum as a random process with continuous state space (i.e., load extrema are not discrete) and more flexible autocorrelation assumption. Frequency domain-based methods characterize loading with power spectral density, spectral moments and bandwidth parameters, and these characteristics are related directly to fatigue damage estimation (Tovo, 2002; Benasciutti & Tovo, 2005; Benasciutti & Tovo, 2007). The application of frequency domain-based methods to fatigue damage prognosis is not straightforward since prognosis-related issues, such as risk assessment and management, inspection and maintenance scheduling, operational decision-making, etc., are mainly defined in the time domain. The autoregressive moving average (ARMA) method is based on time series analysis and characterizes the fatigue load spectrum in the time domain. Available studies identify the order of ARMA models based on some criteria, and then estimate the value of the unknown parameters of models using time series data. The parameters are assumed to be deterministic and the variability and the uncertainty of loading are represented by a random noise term (Leser, Thangjitham, & Dowling, 1994; Leser, Juneja, Thangjitham, & Dowling, 1998). Several important issues remain unsolved. (1) It is unclear what order of model should be selected when several model orders have similar performance under identification criteria, and sometimes it is desired to incorporate multiple competing models. (2) The variability in loading comes from various environmental factors and the mechanisms underlying the load spectrum can be complicated, and hence it may not be appropriate to lump all the variability into one single noise term. (3) The data used to estimate the ARMA model coefficients may not be sufficient and cause additional

uncertainty. (4) Rigorous model validation is desired before the ARMA method is applied to prognosis.

It should be noted that all of the three aforementioned methods are applicable for stationary load spectra, i.e., the statistics of loading are assumed to be constant with respect to time. In practice, the loading condition may be non-stationary. Leser et al. (Leser, Thangjitham, & Dowling, 1994; Leser, Juneja, Thangjitham, & Dowling, 1998) used a truncated Fourier series to account for the non-stationary part of the loading history and model the stationary part with ARMA models. The Fourier series fitted from a time series data is periodic with the length of the data as period, but the real load history may not be periodic. Therefore, a more general modeling framework that accounts for the non-stationary load spectrum is desired.

The first part of this paper provides a brief review of stochastic characterization and reconstruction of fatigue load history using the aforementioned methods, i.e., rainflow counting, the Markov chain method, and the ARMA method. Further, a probabilistic weighting method is applied to the ARMA model to incorporate multiple competing models. The coefficients of ARMA models are assumed to be random variables with unknown probability distributions to represent the variability of loading and the uncertainty from sparse data. Combining the probabilistic weights and the random coefficients, the ARMA model of fatigue load history is formulated.

In the second part of this paper, a continuous model updating framework based on usage monitoring data of load amplitudes is proposed to account for the time-variant features of the load history. Direct updating of the characteristic matrices is applied to the rainflow counting method and the Markov chain method. A Bayesian approach is used to update the probabilistic weights and the coefficients of ARMA model. The relation between updating period and the accuracy of model predictions is evaluated quantitatively. Further, a quantitative model validation metric, namely the Bayes factor, is proposed to assess the validity of ARMA model predictions with respect to usage monitoring data. The proposed techniques are illustrated using numerical examples.

2 STOCHASTIC CHARACTERIZATION OF FATIGUE LOAD SPECTRUM

Section 2 presents three different methods to characterize fatigue load spectrum based on available load amplitude data. Based on the models constructed, random samples of the anticipated load spectrum can be generated and used in stochastic fatigue damage prognosis.

2.1 Rainflow counting method and stochastic reconstruction

Of the well established counting methods, the two-parameter rainflow counting method has the greatest significance for fatigue crack growth analysis (Heuler & Klatschke, 2005) as it fully captures the basic damaging content (number, amplitude, and mean values) contained within the load history, and can be used for uncertainty quantification of the variable amplitude load spectrum.

Following a certain set of rules (ASTM, 2005), the rainflow counting method extracts and counts cycles of various amplitudes and mean values, leaving only a residue behind. These load cycles are considered to be the basic elements of a load sequence. The final counting result is contained in a matrix A of size $k * k$, in which the element a_{ij} gives the number of counted cycles from load level i to load level j , and k is a user defined number of load discretization levels, usually set to 32, 64, or 128 depending on accuracy and computational efficiency desired (Amzallag, Gerey, Robert, & Bahuau, 1994).

Although cycle counting methods have typically been viewed as deterministic methods for characterizing load histories, the obtained results from such methods can be easily transformed for use within a statistical framework. This process can be accomplished by fitting the elements in the rainflow matrix to a joint distribution. Both the "from" load level and "to" load level as determined from the rainflow counting procedure are considered as random variables, say x and y respectively. Assuming that the rainflow counting results are discrete values, the joint probability mass function (PMF) of x and y is denoted as $p(x,y)$ and the joint cumulative distribution function (CDF) $F(x,y)$ given by (Haldar & Mahadevan, 2000):

$$F(x,y) = \sum_{x_i < x} \sum_{y_j < y} p(x_i, y_j) \quad (1)$$

The variables x and y can also be approximated as continuous variables with their joint PDF denoted by $f(x,y)$ and the joint CDF $F(x,y)$ given by:

$$F(x,y) = P(X \leq x, Y \leq y) = \int_{-\infty}^x \int_{-\infty}^y f(u,v) dv du \quad (2)$$

This idea allows the extension of the well understood deterministic rainflow counting procedure to statistical analysis and quantification of the load history, which is of importance in fatigue crack growth analysis. By using the rainflow counting procedure in combination with statistical distribution fitting, a complex load history can be easily broken down into its elementary cycles and efficiently represented by a joint probability density function.

The basic objective of the stochastic rainflow reconstruction algorithm is to create a systematic method to reconstruct a load history given a rainflow

matrix and its residual. Dressler et al. (Dressler, Hack, & Krüger, 1997) presented an algorithm for reconstruction such that an optimal randomization of the reconstructed series is attained. Rainflow reconstructions are based on the idea of extracting cycles from the rainflow matrix and placing them in valid locations in the history under construction. Several rules are defined to ensure that cycles are inserted within the residual in such a way as to yield a similar rainflow matrix as the original signal (Khosrovaneh & Dowling, 1990). Reconstruction is performed in such a way that fatigue cycles are reinserted into the residual in the order of their respective amplitudes, with largest amplitudes inserted first. For each cycle within the rainflow matrix, all of the possible locations for reinsertion are determined, each is given an equal probability, and then a sample location is randomly generated. The cycle is then deleted from the rainflow matrix, and the cycle with the next largest amplitude is considered. This process is repeated until the rainflow matrix is empty and all cycles have been reinserted into the residual. Numerous random sequences can be generated in this manner, and the distribution of life can be estimated based on sequence effects.

Instead of calculating the fatigue life of a component based on a single load sequence, stochastic reconstruction allows for statistical evaluation of the fatigue life based on numerous load sequences that have the same statistical properties as the original spectrum. It should be noted that this method assumes that the original spectrum is representative of the typical load spectrum experienced by the component since all reconstructions are based on the rainflow matrix calculated from the original signal.

2.2 Markov chain method and transition probability matrix

For a realistic loading history, not only the load amplitude at a certain time instant is random, the load amplitudes at adjacent time instants may also be correlated, e.g., the amplitude at time instant T_k can affect the amplitude at time instant T_{k+1} . Given this assumption, fatigue loading history with m discrete load levels is modeled as a discrete time Markov chain $\{X_n\}$, which is a Markov stochastic process whose state space (the set of discretized load levels) is a finite set, and for which n is a discrete time instant ($n = T_0, T_1, T_2, \dots$) (Karlin, 1966). Let event $E_{k,i}$ denote that the loading amplitude at time instant T_k is equal to load level i , and let event $E_{k+1,j}$ denote that the loading amplitude at T_{k+1} is equal to load level j . A one-step transition probability $P_{i,j}^{k,k+1}$ between $E_{k,i}$ and $E_{k+1,j}$ is defined as the probability of $E_{k+1,j}$, given $E_{k,i}$, i.e.,

$$P_{i,j}^{k,k+1} = \Pr(E_{k+1,j} | E_{k,i}) \quad (3)$$

With a further assumption that the one-step transition probability is independent of the time instants, i.e., the transition probability between $E_{k,i}$ and $E_{k+1,j}$ depends on i and j only, a stationary Markov chain transition probability matrix is constructed as:

$$\mathbf{P} = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,m} \\ P_{2,1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ P_{m,1} & \cdots & \cdots & P_{m,m} \end{bmatrix} \quad (4)$$

where $P_{i,j}$ is the one-step stationary transition probability between $E_{k,i}$ and $E_{k+1,j}$, and it holds the same value for all time instants T_k .

Note that fatigue load spectrum is a series of extreme points, i.e., it is formed by minimum – maximum – minimum – ..., etc. Due to this cyclic feature, the transition matrix \mathbf{P} is split into two triangular matrices \mathbf{P}^u and \mathbf{P}^l , as follows:

$$\mathbf{P}^u = \begin{bmatrix} 0 & P_{1,2} & \cdots & P_{1,m} \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & P_{m-1,m} \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \quad (5)$$

$$\mathbf{P}^l = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ P_{2,1} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ P_{m,1} & \cdots & P_{m,m-1} & 0 \end{bmatrix}$$

The elements of the upper triangular matrix \mathbf{P}^u are the transition probabilities from minima to maxima, whereas the elements of the lower triangular matrix \mathbf{P}^l are the transition probabilities from maxima to minima.

Given a load spectrum with discrete load levels from time T_0 to T_e , the element of the stationary Markov chain transition probability matrix $P_{i,j}$ can be estimated using the number of occurrences that the event $E_{k,i}$ is followed by the event $E_{k+1,i}$, i.e.,

$$C_{i,j} = \sum_{k=0}^{e-1} I_{i,j}(k) \quad (6)$$

$$P_{i,j} = \begin{cases} C_{i,j} / \sum_{j=i+1}^m C_{i,j} & i < j, P_{i,j} \in \mathbf{P}^u \\ C_{i,j} / \sum_{j=1}^{i-1} C_{i,j} & i > j, P_{i,j} \in \mathbf{P}^l \end{cases} \quad (7)$$

where $I_{i,j}(k)$ is an indicator function:

$$I_{i,j}(k) = \begin{cases} 0 & \text{if } E_{k,i} \text{ is followed by } E_{k+1,j} \\ 1 & \text{if } E_{k,i} \text{ is not followed by } E_{k+1,j} \end{cases} \quad (8)$$

Note that the matrix \mathbf{C} formed by $C_{i,j}$ is also a counting matrix but with a simple counting rule different from rainfall counting. Once the transition matrix is obtained, random samples of loading history can be conveniently generated from a given initial extreme point. For example, if the load amplitude at the current time instant is equal to level i , the amplitude at the next time instant can be randomly generated based

on the i^{th} row of the transition probability matrix using sampling techniques.

2.3 ARMA process loading

2.3.1 Autoregressive moving average (ARMA) model

The ARMA model is a mix of the autoregressive (AR) and moving average (MA) models. It is widely used in time series analysis for its flexibility. There is no specific pattern assumption in the ARMA model except for its order. Only the information from time series data is used to construct the model.

The autoregressive (AR) model represents the value at the current time instant in terms of the values at the previous time instants. Hence, it is capable of capturing the autocorrelation between time series. A p^{th} order AR model can be written as:

$$Y_t = \varphi_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \varepsilon_t \quad (9)$$

where Y_t is the value at time instant t ; Y_{t-i} is the value at time instant $t-i$ (there are i time lags before t); φ_i is the coefficient of the AR model; ε_t is the random noise term with respect to time instant t .

The moving average model represents the deviation of the series at the current time instant from its mean value as a linear combination of errors in the past time instants. A q^{th} order MA model can be written as:

$$Y_t = \mu + \varepsilon_t - \omega_1 \varepsilon_{t-1} - \omega_2 \varepsilon_{t-2} - \cdots - \omega_q \varepsilon_{t-q} \quad (10)$$

where μ is a constant which can be considered as the mean value over time; ε_{t-i} is the random noise term at time instant $t-i$.

Combining a p^{th} order AR model and a q^{th} order MA model, an ARMA(p, q) model is obtained as:

$$Y_t = \varphi_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + \cdots + \varphi_p Y_{t-p} + \varepsilon_t - \omega_1 \varepsilon_{t-1} - \omega_2 \varepsilon_{t-2} - \cdots - \omega_q \varepsilon_{t-q} \quad (11)$$

2.3.2 Identification of ARMA model

The first step to build an ARMA model is to identify its orders p and q . The sample autocorrelation function and the sample partial autocorrelation function of the stationary time series data obtained from differentiation is used for this purpose.

The autocorrelation function (ACF) for a stationary time series Y with mean μ and standard deviation σ is defined as:

$$\rho_k = \frac{E[(Y_t - \mu)(Y_{t+k} - \mu)]}{\sigma^2} \quad (12)$$

where the operator E refers to the expected value; ρ_k is the autocorrelation function for time lag k , i.e., the correlation between Y_t and Y_{t+k} .

The partial autocorrelation function (PACF) at time lag k is defined as a measure of the correlation between Y_t and Y_{t+k} without accounting for the effects of the values at intermediate time instants ($Y_{t+1}, \dots, Y_{t+k-1}$).

Derivation and estimation of the partial autocorrelation function are given in (Box, Jenkins, & Reinsel, 1994).

It is known that each ARMA model has a unique pattern for its ACF and PACF (Hanke & Wichern, 2005). However, subjectivity is involved while visually comparing the sample ACF and PACF with the theoretical values. One way to address the identification problem is to first select an initial model tentatively, and then the parameters and residuals associated with the selected model are estimated using the least square method. Hereafter, some statistics or criteria are used to check for adequacy; if the tentative model is shown to be inadequate to represent the data, it may be replaced by another model. The Ljung-Box Q statistic (Ljung & Box, 1978), which is a function of the residual autocorrelations and is approximated as a chi-square random variable, is used here to check the adequacy of the tentative model. The formula for Q is:

$$Q_m = n(n+2) \sum_{k=1}^m \frac{r_k^2}{n-k} \quad (13)$$

where Q_m is a chi-square variable with $m-r$ degrees of freedom, and r is the number of the estimated parameters in the ARMA model; r_k is the residual autocorrelation at time lag k ; n is the number of residuals; and m is the number of time lags considered in this test. If the p -value, which is equal to $(1 - \text{the cumulative probability of } Q \text{ evaluated at } Q_m)$, is not large enough, this tentative model is rejected.

2.3.3 Uncertainty in the usage of ARMA model

In previous studies (Leser, Thangjitham, & Dowling, 1994; Leser, Juneja, Thangjitham, & Dowling, 1998), the coefficients of ARMA model are treated as deterministic and estimated through moment estimator. The inherent variability of loading amplitude and uncertainty from data are incorporated into the single noise term ε_t , which is assumed as an independently and identically distributed random process with zero mean and constant variance. Mechanical components usually work under complicated operating environments and many factors contribute to the variability of loading amplitudes. The uncertainty due to limited amount of data can also be significant. Therefore, a single noise term is not always sufficient. To accurately capture the aforementioned variability and uncertainty, the parameters - $\boldsymbol{\varphi}$ and $\boldsymbol{\omega}$ - of ARMA model, along with the noise term ε_t , can be assumed to be random variables. At the beginning of prognosis, if no information about the probability distributions of $\boldsymbol{\varphi}$ and $\boldsymbol{\omega}$ is available, a uniform prior distribution may be assumed and further calibrated by usage monitoring data based on Bayes theorem, which will be explained in detail in Section 3.2.

Besides inherent variability of loading amplitudes and uncertainty from data, additional uncertainty arises in the selection of appropriate ARMA model, which

may be referred to as model uncertainty. In principle, there is only one correct model for a particular problem (Soares, 1997). However, it may not be obvious that which model is the correct one when multiple competing models are available. The tentative model identification method with the Q statistics illustrated in Section 2.3.2 can help eliminate models that are insufficient to represent the data, and there may still be several competing models left. The risk of choosing a single incorrect model may be minimized by considering several possible models. A straightforward way to incorporate multiple models is to assign a probabilistic weight to each of the competing models (Zhang & Mahadevan, 2000). The probabilistic weight P_{M_i} for the model M_i represents the probability of the model M_i being correct. Combining the uncertainty in the ARMA model parameters and the probabilistic weights, the probability density function of the fatigue loading amplitude at time t is:

$$f(Y_t) = \sum_{i=1}^n P_{M_i} \int F * f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) f(\mathbf{Y}_{-t}) d\boldsymbol{\varphi}^i d\boldsymbol{\omega}^i d\mathbf{Y}_{-t} \quad (14)$$

$$F = f(Y_t | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t})$$

where \mathbf{Y}_{-t} is the vector containing the model outputs in the previous time steps $Y_{t-1}, Y_{t-2}, \dots, Y_{t-p}$, and M_i 's ($i = 1, 2, \dots, n$) are the competing models. $f(Y_t | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t})$ is the conditional probability density function of loading amplitude Y_t for a given ARMA model M_i and its associated parameters $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, which can be derived from Eq. 11. $f(\mathbf{Y}_{-t})$ is the joint probability density distribution of loading amplitudes in the previous time steps, which is obtained using Eq. 14 in the previous time steps. $f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i)$ is the probability density function of $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, which can be assumed as uniform at the beginning of prognosis if no prior information is available. Similarly, all the values of probabilistic weights P_{M_i} 's can be assumed as $1/n$ if no information is available to support any single model. These prior assumptions can be calibrated based on usage monitoring data, as will be discussed in Section 3.2.

Once the probability distribution of loading amplitude with respect to time instant t is obtained, samples of future anticipated loading can be generated and applied in probabilistic fatigue prognosis.

3 STATISTICAL UPDATING OF LOAD MODELS BASED ON USAGE MONITORING DATA

The samples of anticipated load spectrum required in stochastic fatigue prognosis can be generated through the three methods presented in Section 2 based on the available data. An assumption underlying the application of these methods is that the available data fully represent the load spectrum and provides sufficient information to predict future loading. This

assumption is challenged when the available data is limited and significant uncertainty exists. Further, the characteristics of loading may vary gradually with time, due to the change of operating environments of mechanical components. A continuous updating framework incorporating the load modeling methods is therefore proposed in this section based on usage monitoring data. Section 3.1 presents a direct updating scheme for both the rainflow counting method and the Markov chain method. Section 3.2 presents a Bayesian approach for updating the ARMA model.

3.1 Direct updating of the characteristic matrix

Both the rainflow counting method and the Markov chain method characterize fatigue load spectrum with a single matrix. In the rainflow counting method, the counting matrix stores the number of cycles from one load level to another load level; in the Markov chain method, the transition probability matrix stores the transition probability from one load level to another load level. As mentioned in Section 2.1 and 2.2, the elements of these two characteristic matrices are obtained based on the available load amplitude data, and samples of load spectrum can be generated. Once a new set of data is collected, the rainflow counting method and the Markov chain method are applied to obtain updated characteristic matrices. If the pattern of new data is different from the previous data, it can then be incorporated into the updated characteristic matrices. For the rainflow counting method, a new counting matrix can be obtained from the new data set, and it can be added directly to the previous characteristic matrix to obtain an updated matrix. Similarly for the Markov chain method, a new \mathbf{C} matrix with elements C_{ij} as shown in Eq. 6 can be derived from the new data set, and then the addition of the new \mathbf{C} matrix and the previous \mathbf{C} matrix forms an updated \mathbf{C} matrix. An updated transition probability matrix can be obtained from this updated \mathbf{C} matrix as shown in Eq. 7. The updated characteristic matrices of these two methods can then be used to generate samples of load spectrum for the next period of prognosis.

The accuracy of simulated load histories from these two methods can be evaluated by an average square error (ASE):

$$ASE = \left[\sum_{t=1}^T (Y_t - Y_{Dt})^2 \right] / T \quad (15)$$

where Y_t is the simulated load amplitude at time t , whereas Y_{Dt} is the load amplitude data at time t ; T is the total length of time. If the value of ASE is small, it indicates that the simulated load history is close to the real data.

3.2 Bayesian updating of the ARMA model

3.2.1 Model Calibration based on the Bayes Theorem

Considering one of the competing ARMA models M_i , with the associated parameters $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, the load amplitude Y_t is predicted as:

$$Y_t = M_i(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, t, \mathbf{Y}_{-t}) \quad (16)$$

Note that the model M_i contains uncertainty from the random noise terms $\varepsilon_t, \varepsilon_{t-1}, \dots, \varepsilon_0$, in addition to the variability in its parameters. The noise terms are assumed as normal random variables with zero mean and variance σ_ε^2 . The variance σ_ε^2 can be estimated by comparing the model predictions with available data set \mathbf{D} , as follows:

$$\sigma_\varepsilon^2 = \frac{1}{n-1} \sum_k^n [Y_{Dk} - Y_{ik}]^2 \quad (17)$$

where Y_{Dk} is the k^{th} element in the data set, and Y_{ik} is the corresponding prediction of the model M_i with given values of parameters $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$; n is the number of the data points considered.

The conditional probability density function $f(Y_t | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t})$ of the model output Y_t of M_i for given values of $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$ can be constructed based on Eqs. 16-17, and the probability distributions of the noise terms. Monte Carlo simulation is used in this paper to estimate $f(Y_t | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t})$.

Assuming a joint prior distribution $f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i)$ for $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, the calibrated distribution of $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$ given a collection of data \mathbf{D} , $f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i | \mathbf{D})$, is obtained as:

$$f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i | \mathbf{D}) = \frac{L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i)}{\int L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) d\boldsymbol{\varphi}^i d\boldsymbol{\omega}^i} \quad (18)$$

where the likelihood function of $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, $L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i)$, is the probability of observing the collected data for given values of $\boldsymbol{\varphi}^i$ and $\boldsymbol{\omega}^i$, which is calculated as:

$$L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) = \int f(\mathbf{Y}_D | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t}) f(\mathbf{Y}_{-t}) d\mathbf{Y}_{-t} \quad (19)$$

By assuming the data points are independent of each other, Eq. 19 can be further written as:

$$L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) = \prod_{k=1}^n \int f(Y_{Dk} | M_i, \boldsymbol{\varphi}^i, \boldsymbol{\omega}^i, \mathbf{Y}_{-t}) f(\mathbf{Y}_{-t}) d\mathbf{Y}_{-t} \quad (20)$$

The probabilistic weight of M_i , i.e., the probability of M_i being the correct model, can be calibrated using Bayes theorem as:

$$P(M_i | \mathbf{D}) = \frac{L(M_i) P(M_i)}{\sum_{j=1}^m L(M_j) P(M_j)} \quad (21)$$

where $P(M_i)$ and $P(M_i | \mathbf{D})$ are the prior weight and updated weight, respectively; $L(M_i)$ is the likelihood function of M_i , that is, the probability of observing the data with the assumption that M_i is the correct model. $L(M_i)$ is calculated as:

$$L(M_i) = \iint L(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) f(\boldsymbol{\varphi}^i, \boldsymbol{\omega}^i) d\boldsymbol{\varphi}^i d\boldsymbol{\omega}^i \quad (22)$$

3.2.2 Continuous Bayesian updating of the ARMA model

Model calibration based on the Bayes theorem can be applied to the ARMA model continuously with usage monitoring data. The updated model can then represent the pattern of the latest data without losing information contained in the previous data sets. In addition, Bayesian updating can reduce the uncertainty in the model coefficients and the selection of models as more data are used. The continuous Bayesian updating can be implemented following five steps as shown below:

(1) At the beginning of prognosis, by using the initial set of data, the ARMA models satisfying the Q statistic based-criteria are identified as competing models. If no prior information about the probability distribution of the corresponding model parameters, uniform distributions are first assumed as the priors. Similarly, if no model is preferable from the prior knowledge, they are assumed to be equally weighted.

(2) The probability distributions of model parameters and probabilistic model weights are calibrated using Eq. 18 and Eq. 21 as mentioned in Section 3.2.1.

(3) With the estimated distributions of model parameters and weights, the probability distribution of predicted loading amplitude with respect to time is obtained using Eq. 14. Samples of the load spectrum are then generated with sampling techniques and applied in fatigue prognosis.

(4) After a new set of usage monitoring data is collected, Step (2) is again conducted by assuming the previously estimated ARMA model parameter distributions and model weights as priors.

(5) Repeat Steps (2) to (4), until the end of the prognosis.

In the above continuous updating procedure, the length of updating period remains unclear. A shorter period length means usage monitoring data is retrieved more frequently and so is the updating. The increased data transmission activities will lower the battery life of the monitoring device, and more updating will increase the computational effort. It is desired to find an optimum period that balances prediction accuracy and efficiency. Therefore, it is desired to investigate the effect of the updating period length on the accuracy of the ARMA model prediction.

Note that the output of the ARMA model is a random process indexed by time, and hence two quantitative statistical metrics – mean square error MSE_p of mean prediction with respect to load history data, and the width W_p of the 95% prediction bounds are used to evaluate the accuracy of model output for a selected updating period:

$$MSE_p = (E(Y_t) - Y_{Dt})^2 \quad (23)$$

$$W_p = F_{Y_t}^{-1}(0.975) - F_{Y_t}^{-1}(0.025) \quad (24)$$

where $E(Y_t)$ is the mean prediction of ARMA model at time t , whereas Y_{Dt} is the load amplitude data at time t ; $F_{Y_t}^{-1}$ is the inverse cumulative probability function of Y_t , e.g., $F_{Y_t}(F_{Y_t}^{-1}(0.975)) = 0.975$. If the value of MSE_p is small, the prediction of ARMA model is close to the real data, i.e., the prediction is accurate with the corresponding updating period. If the value of W_p is small, the uncertainty in the prediction of ARMA model with the corresponding updating period is also small.

3.2.3 Validation of the ARMA model

In fatigue damage prognosis, the ARMA model is used for the prediction of future loading, and it is desired to validate the prediction. Validation involves comparison of model prediction with experimental data (Rebba, Mahadevan, & Huang, 2006). A quantitative validation metric based on the Bayesian approach, namely the Bayes factor (Jeffreys, 1961), is proposed here to validate the ARMA model. Considering a model M , when data \mathbf{D} is observed, the Bayes factor is defined as a likelihood ratio:

$$B = \frac{P(\mathbf{D} | M \text{ is correct})}{P(\mathbf{D} | M \text{ is not correct})} \quad (25)$$

If the Bayes factor is greater than 1 then it indicates that the data supports the model M , otherwise it indicates that the data does not support the model M .

It has been shown that the Bayes factor can be expressed using prior and posterior probability density function (PDF) values at the model prediction when there is no prior information about model validity (Rebba, Mahadevan, & Huang, 2006). Thus,

$$B(Y_t) = \frac{f(Y_t | Y_{Dt})}{f(Y_t)} \quad (26)$$

where Y_{Dt} is the data collected at time t . The prior PDF for ARMA model prediction is computed using Eq. 14. The posterior PDF is computed using Bayes theorem as:

$$f(Y_t | Y_{Dt}) = \frac{L(Y_t)f(Y_t)}{\int L(Y_t)f(Y_t)dY_t} \quad (27)$$

By assuming that the data is collected with measurement noise, a normal random variable with zero mean and variance σ_M^2 , the likelihood function $L(Y_t)$ is:

$$L(Y_t) = P(Y_{Dt} | Y_t) = \frac{1}{\sqrt{2\pi\sigma_M^2}} \exp\left\{-\frac{1}{2} \frac{[Y_{Dt} - Y_t]^2}{\sigma_M^2}\right\} \quad (28)$$

The $B(Y_t)$ obtained from Eq. 26 is the value of the Bayes factor when the model prediction equals Y_t at time t . Further, the Bayes factor can be computed as a function of time as:

$$B(t) = \int B(Y_t)f(Y_t | \mathbf{D})dY_t \quad (29)$$

Further, the degree of confidence in the model prediction C can be measured as:

$$C(t) = \frac{B_t}{1+B_t} * 100\% \quad (30)$$

3.3 Summary

A continuous model updating framework is developed in this section to capture the non-stationary pattern of the fatigue load spectrum. Direct updating of characteristic matrices is applied to the rainflow counting method and the Markov chain method. A Bayesian updating approach is applied to the ARMA model through calibrating the probability distributions of the model coefficients and the values of the probabilistic weights. The uncertainty in the ARMA model can be reduced as more data are used. The effect of updating period to the accuracy of model prediction can be investigated using two quantitative metrics. A quantitative validation metric, the Bayes factor, is proposed to evaluate the validity of model predictions.

4 NUMERICAL EXAMPLE

A scaled helicopter combat maneuver loading history data including 510 cycles (1020 extrema/turning points) (Khosrovaneh, Dowling, Berens, & Gallagher, 1989) as shown in Fig. 1 is used for the illustration of the rainflow counting and reconstruction method, the markov chain method, and the ARMA model method. It can be observed from the data plot that the load spectrum shows a time-variant pattern, and hence the proposed continuous updating framework is also applied to the aforementioned three methods.

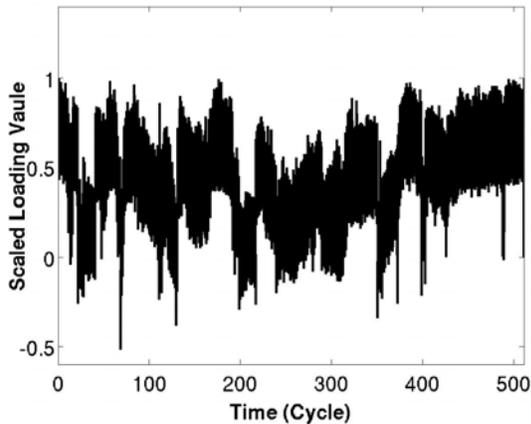


Figure 1: A Scaled Helicopter Combat Maneuver Load History Data

4.1 Rainflow counting, stochastic reconstruction and updating

For the purpose of illustration, two subsets of the original data set are used, 1-250 cycles and 251-500 cycles. The first subset (1-250 cycles) is assumed to be

the data currently available and is used to conduct the initial rainflow counting. A graphical representation of the counting matrix is shown in Fig. 2(a). Samples of simulated load history are generated from the counting matrix using the reconstruction technique introduced in Section 2.1.2, as shown in Fig. 2(b). The samples of load history show random rearrangements of the cycles extracted from the data. For the purpose of prognosis, the generated samples can be used as the prediction for future loading cycles, i.e., load amplitudes during 251-500 cycles, before the new usage monitoring data is collected.

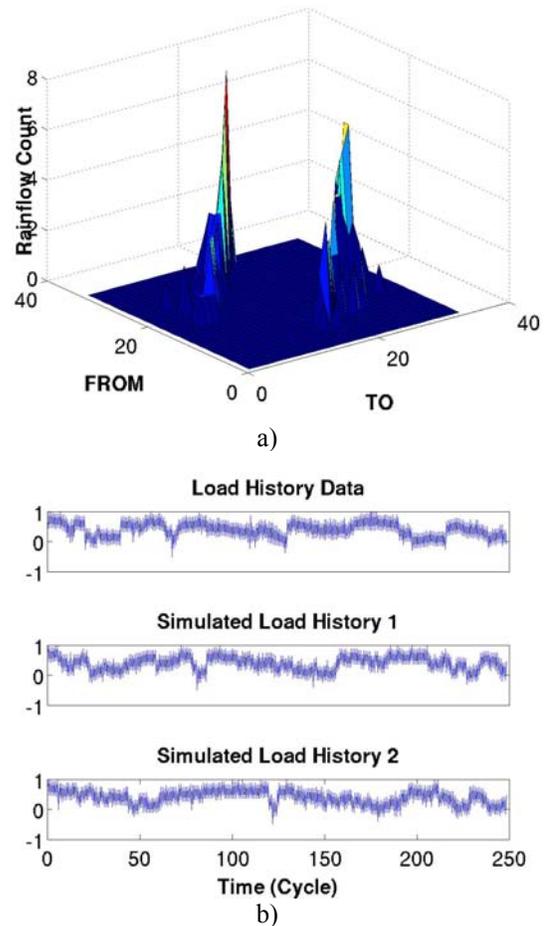


Figure 2: a) Graphical representation of rainflow counting matrix from the first subset of the load history data (1-250 cycles); b) comparison of the load history data and two samples of simulated load history (1-250 Cycles)

The simulated load histories can be quantitatively compared with data by the ASE metric (Eq. 15), as shown in Table 1. “As reconstruction” indicates the simulated load histories are used as reconstruction of the original load spectrum and then are compared with the data used to generate the counting matrix (1-250 cycles in this example). “As prediction” means the

simulated load histories are used for prediction purpose and are compared with the data in the future time interval (251-500 cycles in this example).

Table 1: ASE of simulated load histories (1-250 cycles)

Average square error	Simulated load history 1	Simulated load history 2
As reconstruction	0.0916	0.0927
As prediction	0.0942	0.0904

Consider the second subset of data (251-500 cycles) as the newly collected usage monitoring data, and then the direct updating method presented in Section 3.1 can be applied. First the rainflow counting technique is implemented on the new data and a new cycle counting matrix is obtained. This new matrix is added to the matrix counted from the previous data set (1-250 cycles) and then an updated counting matrix is obtained, as shown in Fig. 3(a). Further, samples of loading history are generated based on the updated counting matrix, and these samples can be again considered as predictions for future loading and used in probabilistic fatigue damage prognosis.

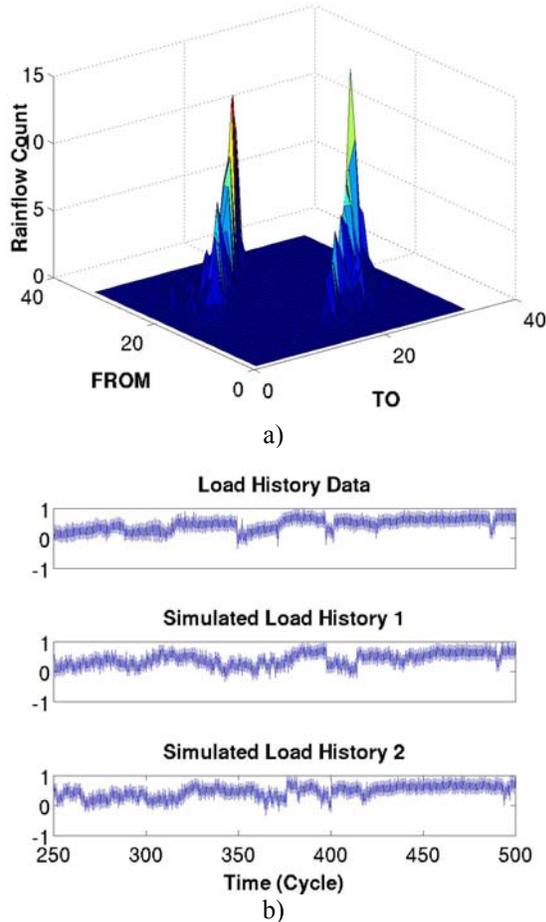


Figure 3: a) Graphical representation of the updated rainflow counting matrix using the second subset of the data; b) comparison of the load history data and two samples of simulated load history (251-500 cycles)

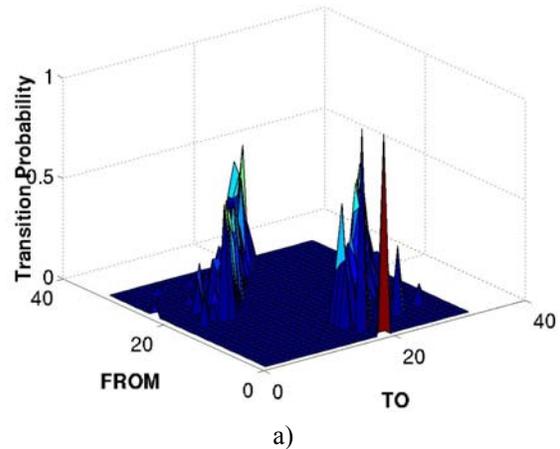
Table 2 shows the average square error (ASE) for two samples of simulated load histories. No data in the future time interval (after the 500th cycle in this example) is available and so the ASE is calculated only for the case that the simulated load histories are used as reconstruction.

Table 2: ASE of simulated load histories (251-500 cycles)

Average square error	Simulated load history 1	Simulated load history 2
As reconstruction	0.0261	0.0310

4.2 Markov chain method

The two subsets of data in Section 4.1 are also used to illustrate the Markov chain method and the updating of the transition probability matrix. The first data subset is assumed as the initially available data set, and the second data subset is the usage monitoring data set obtained later. Following the method presented in Section 2.2, the initial transition probability matrix is estimated using the first data subset and samples of the simulated load spectrum are generated as shown in Figs. 4(a)-(b). The generated samples are considered as the prediction of future loading and used in prognosis for the next time period (251-500 cycles). After a new set of usage monitoring data is obtained (the second data subset), the initial transition probability matrix is updated and then predictions for future loading can be again generated for prognosis, as shown in Figs. 4(c)-(d).



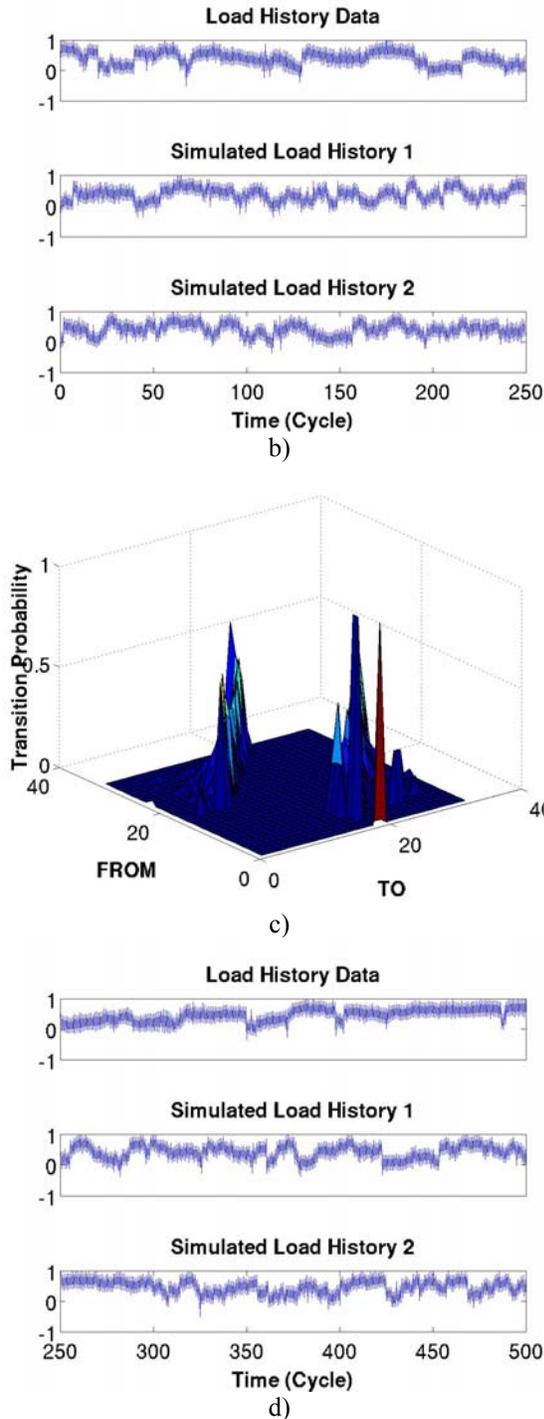


Figure 4: a) Graphical representation of markov chain transition probability matrix using the first subset of the load history data (1-250 cycles); b) comparison of the data and two samples of simulated load history (1-250 cycles); c) Graphical representation of updated Markov chain transition probability matrix using the second subset of the data; d) comparison of data and two samples of simulated load history (251-500 cycles)

Similarly as in Section 4.1, the average square error is calculated to evaluate the accuracy of simulated load histories from the Markov method, as shown in Tables 3 and 4.

Table 3: ASE of simulated load histories (1-250 cycles)

Average square error	Simulated load history 1	Simulated load history 2
As reconstruction	0.0852	0.0695
As prediction	0.0758	0.1044

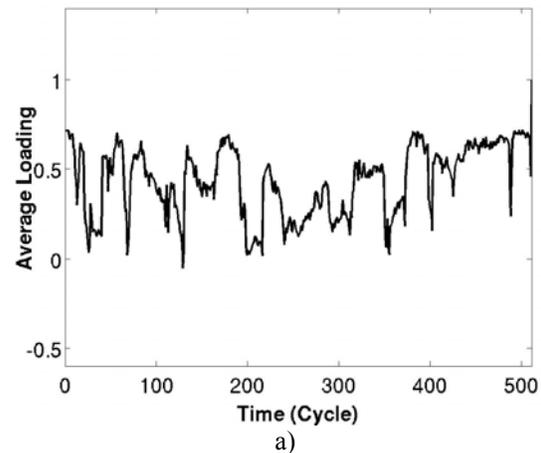
Table 4: ASE of simulated load histories (251-500 cycles)

Average square error	Simulated load history 1	Simulated load history 2
As reconstruction	0.0825	0.0770

4.3 ARMA model method

4.3.1 Partition of data set and initial model identification

The whole data set (510 cycles) is used to demonstrate the extended ARMA model method, the Bayesian approach, and the model verification and validation methodology. Due to the cyclic nature of fatigue loading, the load spectrum is split into two parts, the mean amplitude and the cycle variation, as shown in Fig. 5,



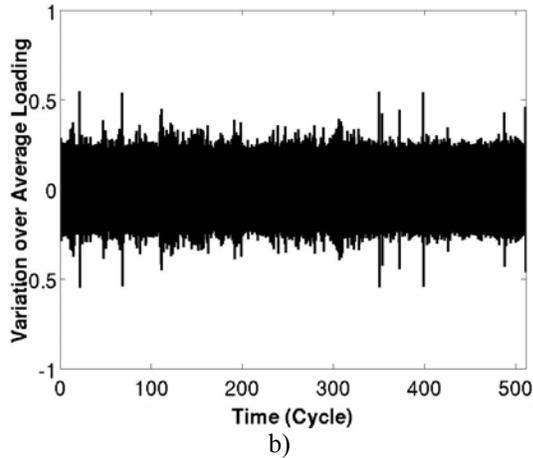


Figure 5: a) The mean amplitude of the load spectrum;
b) the cycle variation of the load spectrum

The sample ACF and PACF of the mean amplitude data, as shown in Figs. 6(a)-(d), suggest that the mean amplitude of the load spectrum can be modeled as an ARMA process, whereas the lack of ACF and PACF in the cycle variation suggest that it can be modeled as a white noise.

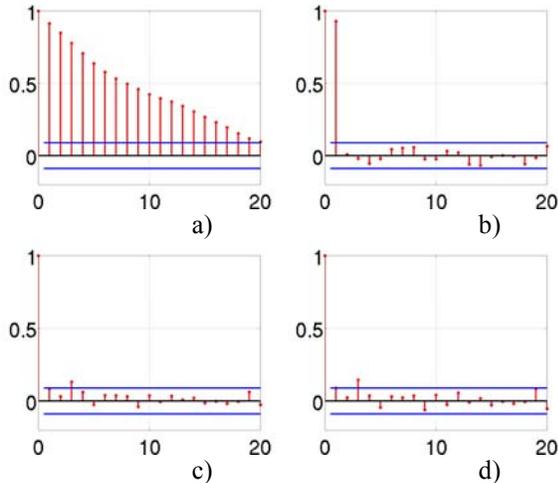


Figure 6: a) Sample autocorrelation function (ACF) of the mean amplitude of the load history data; b) sample partial autocorrelation function (PACF) of the mean amplitude of the load history data; c) sample ACF of the cycle variation of the load history data; d) sample PACF of the cycle variation of the load history data

To illustrate the continuous updating framework, the original data set is divided into several subsets. The first data set (1-250 cycles) is considered as the initially available data, and the following data sets are assumed as the usage monitoring data retrieved subsequently.

The initial data set is used to identify possible ARMA models based on the Q statistics and the associated p -values presented in Section 2.3.2. As shown in Table 5, both ARMA(1,0) and ARMA(2,0) pass the chi-square test since the corresponding p -values are significant. Therefore ARMA(1,0) and ARMA(2,0) are considered as candidate models for the load spectrum.

Table 5: Calculated Q statistics and the associated p -values

	Time Lag	12	24	36
ARMA(1,0)	Ljung-Box Q	10.8	22.8	31.1
	p-value	0.37	0.41	0.61
ARMA(2,0)	Ljung-Box Q	8.45	20.2	28.9
	p-value	0.58	0.57	0.71

4.3.2 Continuous updating on model parameters and probabilistic weights

The parameters of ARMA(1,0) - φ_0, φ_1 , and the parameters of ARMA(2,0) - $\varphi_0, \varphi_1, \varphi_2$ are assumed as random variables. Following the Bayesian approach in Section 3.2.1, the initial probability distributions of these variables can be estimated by combining the likelihood functions from the initial data set and non-informative priors. Probabilistic weights are assigned to ARMA(1,0) and ARMA(2,0) respectively. The initial values of the weights are assumed equal to each other, i.e., the two candidate models are initially assumed to have equal probability of being the correct model for the loading history.

With the usage monitoring data set retrieved in succession, the probability distributions of ARMA model parameters and the probabilistic weights are continuously updated, as presented in Section 3.2.1 and 3.2.2. For the purpose of illustration, the plot of initial probability distributions, updated distributions using the second usage monitoring data set, and the updated distributions using the third usage monitoring data set are shown in Figs. 7(a)-(d). The plot of the updated values of the probabilistic weights is also shown in Fig. 7(e).

As shown in Figs. 7(a)-(d), the widths of the probability distribution functions of the ARMA model parameters shrink gradually with the continuous updating process, i.e., the uncertainty due to sparse data decreases as more data is retrieved. The increasing values of the probabilistic weight for ARMA(1,0) as shown in Fig. 7(e) suggest that ARMA(1,0) obtained increasing support from usage monitoring data during the continuous updating process.

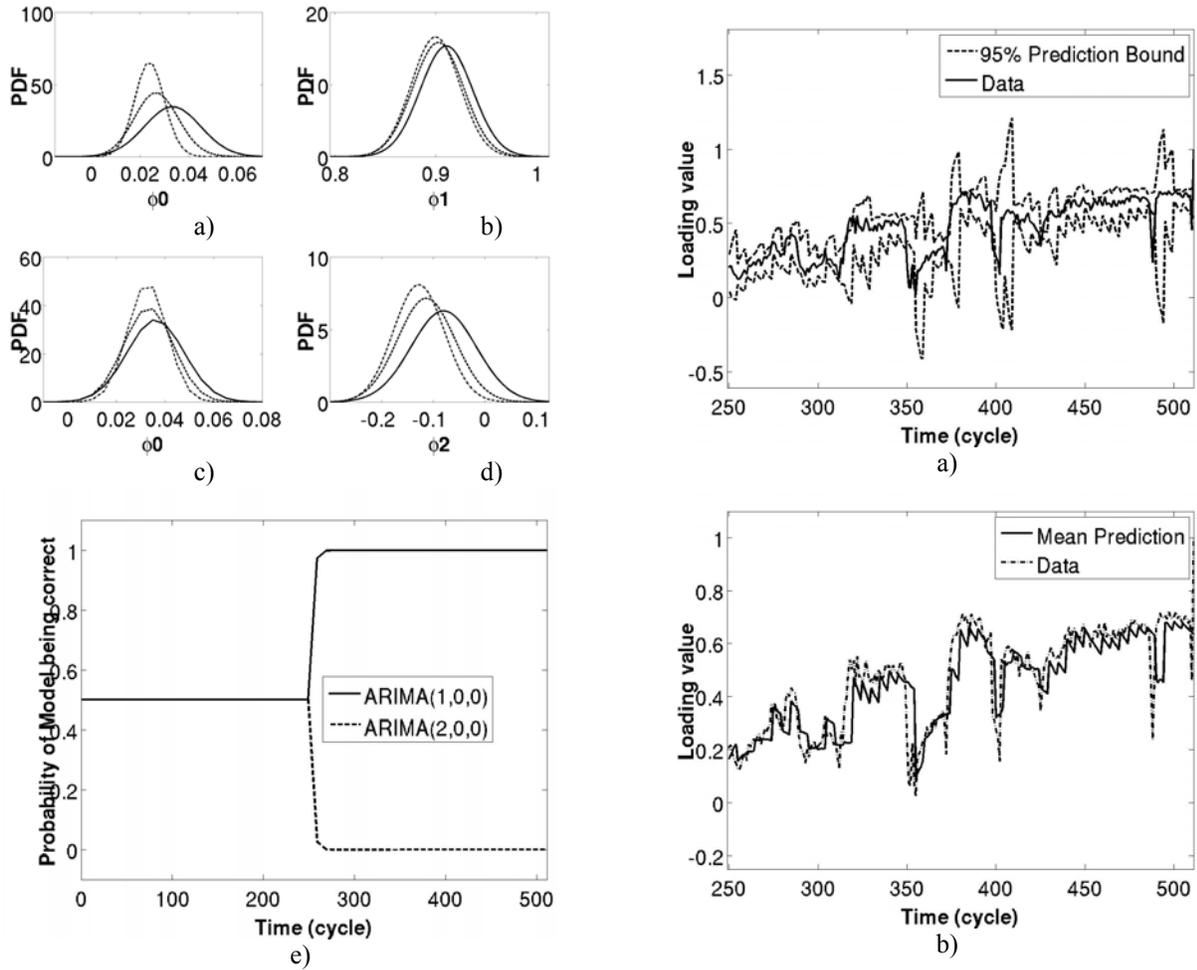
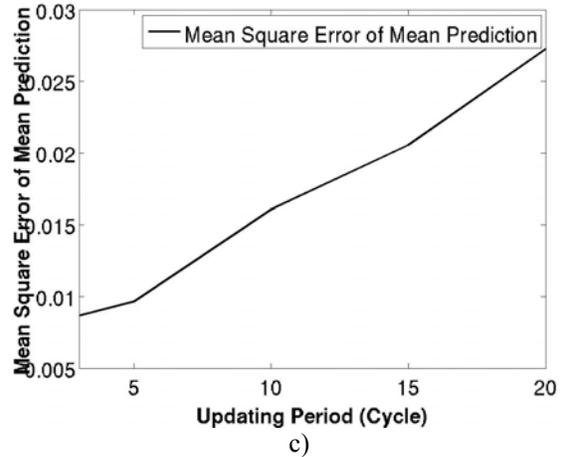


Figure 7: a)-d) Initial probability distribution functions of the ARMA model parameters – ϕ_0 , ϕ_1 of ARMA(1,0), ϕ_0 , ϕ_2 of ARMA(2,0) - and the updated distributions with newly collected data sets; e) the updated values of the probabilistic weights versus time

The two metrics presented in Section 3.2.2, MSE_p of mean prediction with respect to load history data, and the width of 95% prediction bound W_p , are calculated to investigate quantitatively the relationship between the model prediction accuracy and the model updating period. Figs. 8(a)-(b) give a graphical illustration of the two metrics when the updating period is five cycles. Figs. 8(c)-(d) plot the relations between the two metrics and the updating period. It is observed that W_p decreases as the updating period decrease, which suggests that the uncertainty in model predictions can be reduced by more frequent updating; similarly, MSE_p also decreases as the updating period decreases, which suggests that the accuracy of model prediction can be improved by more frequent updating. It is also shown that the model prediction can capture the time-variant feature of data by continuous updating.



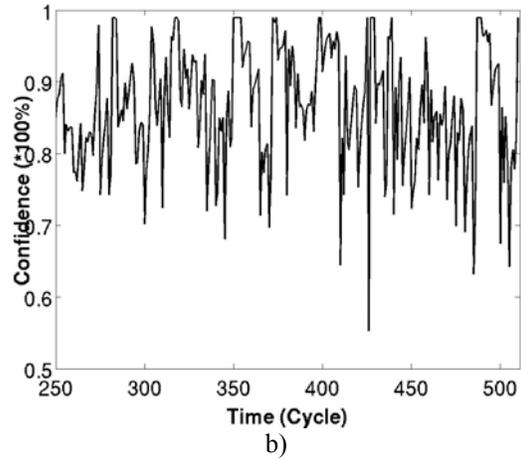
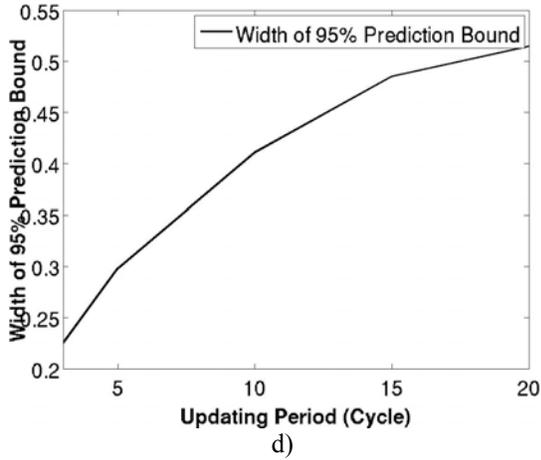


Figure 8: a)-b) Prediction bound and mean prediction versus data when the updating period is 5 cycles; c) MSEp of mean prediction versus the updating period; d) width of 95% prediction bound W_p versus the updating period

Figure 9: a) The Bayes factor versus time; b) the confidence in prediction versus time

4.3.3 Model validation

The predictions from the ARMA model method based on the continuous updating are validated using the Bayes factor presented in Section 3.2.3. By assuming that the measurement noise follows normal distribution with zero mean and standard deviation equals to 0.001, the Bayes factor and the degree of confidence in prediction are calculated using Eqs. 29-30. As shown in Figs. 9(a)-(b), the Bayes factors are much higher than unity, and the degrees of confidence are therefore much higher than 50%. This suggests that the model predictions have a good support from the corresponding data.

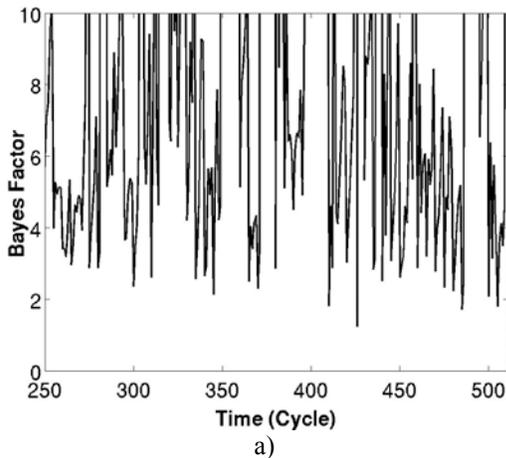
4.4 Comparison of the three methods

The performance of the three methods on future loading prediction can be evaluated using the mean square error metric (Eq. 23). The mean prediction $E(Y_i)$ in Eq. 23 can be estimated by Monte Carlo simulation. The data is partitioned in the same way as Section 4.3.1. All the three methods generate loading prediction along with a 5-cycle updating period, i.e., each model is updated every 5 cycles and the models make predictions for loading in the next 5 cycles.

Table 6: MSE of the three methods

	Mean square error
Rainflow	0.0172
Markov	0.0290
ARMA	0.0112

As shown in Table 6, ARMA has the least value of MSE, which indicates the prediction from ARMA method has the highest accuracy in this example.



5 DISCUSSION

Three different methods, namely rainflow counting method, Markov chain method and ARMA model method, to characterize and reconstruct fatigue load spectra for prognosis are reviewed. The ARMA method is extended through random parameters and probabilistic weights to accommodate the inherent variability in loading, the uncertainty due to sparse data, and the uncertainty in model selection. A continuous model updating framework with usage monitoring data is developed, including direct updating of the characteristic matrices for the rainflow counting method and the Markov chain method, and a Bayesian updating approach for the ARMA model method. The

relation between prediction accuracy and updating period is investigated quantitatively. It is shown in Section 4.3.2 that the continuous updating framework helps the ARMA model method to capture the time-variant feature of fatigue loading and also reduce the uncertainty in prediction due to limited amount of data. A rigorous validation metric, the Bayes factor, is used to provide quantitative assessment to model prediction. It is shown in Section 4.3.3 that the extended ARMA model method with the continuous updating framework performs well in the numerical example, as the Bayes factor and the degree of confidence in prediction suggest that the data strongly supports the model prediction. A comparison based on mean square error measure between these three methods also indicates that the prediction from the ARMA method has the highest accuracy in this example. Although the consideration of uncertainty in model parameters and model form in the extended ARMA method, along with the Bayesian updating approach, increase the number of function calls by the magnitude of 100 if Monte Carlo simulation technique is used, it is usually affordable since the ARMA model takes very little time to run.

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NOMENCLATURE

$f(x \mathbf{D})$	conditional probability density function of random variable x given data \mathbf{D}
ARMA	autoregressive moving average
ϕ, ω	coefficients of ARMA model
Q	Ljung-Box Q statistic
MSE_p	mean square error of the ARMA model mean prediction
W_p	width of the ARMA model prediction bounds
B	Bayes factor

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