Uncertainty Management in Prognostics

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PHM 2010 tutorial, October 11, 2010

Outline

- Background and introduction
- Uncertainty management in prognostics
  - Model-based fatigue damage prognosis
  - Uncertainty quantification and propagation
  - Information fusion and uncertainty updating
  - Model comparison and validation
- Conclusions

Background of Prognostics

- Prognostics: predict the remaining useful life (RUL) when a component/system will no longer perform a particular function (failure)
- Approaches:

  - Data-driven approach

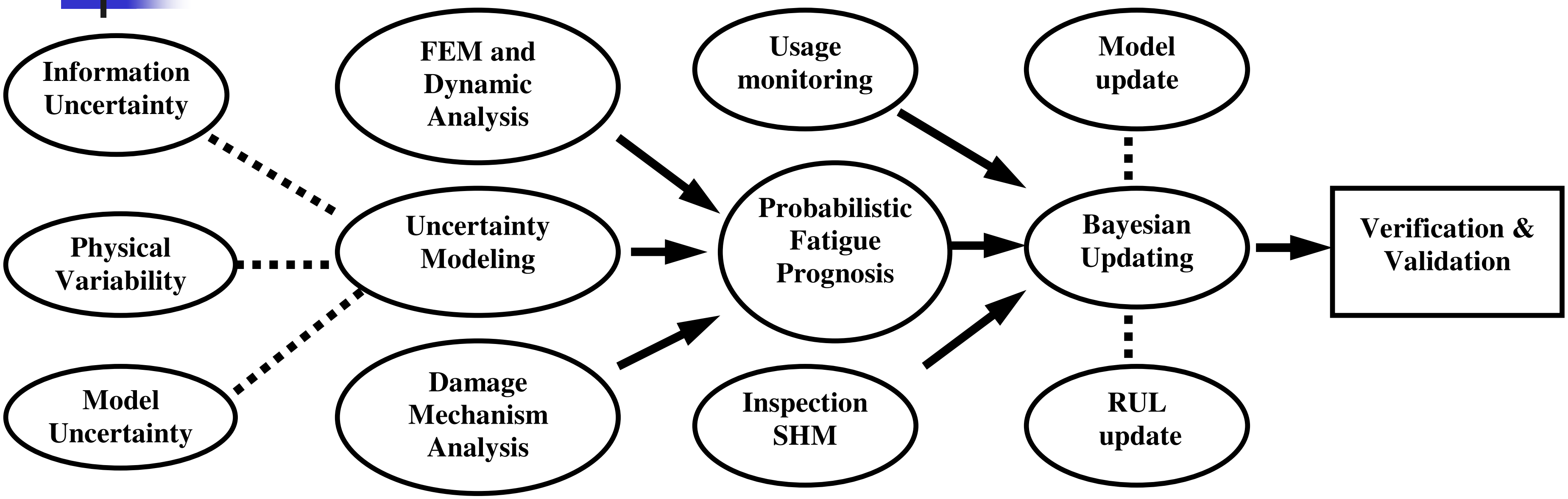
  - Model (physics)-based approach

  - Hybrid approach

Uncertainty management in prognostics - 1

- Both deterministic and probabilistic methods exist for prognostics
- Huge uncertainties associated with some engineering prognostics problems (e.g., fatigue damage prognosis)
- Assist the risk assessment and decision making
- Rigorous information fusion and leveraging different approaches
- Uncertainty reduction in prognostics

Uncertainty management in prognostics - 2



- A sound uncertainty management methodology
  - Mechanism Model
  - Uncertainty Quantification and Propagation
  - Uncertainty Updating
  - Verification and Validation

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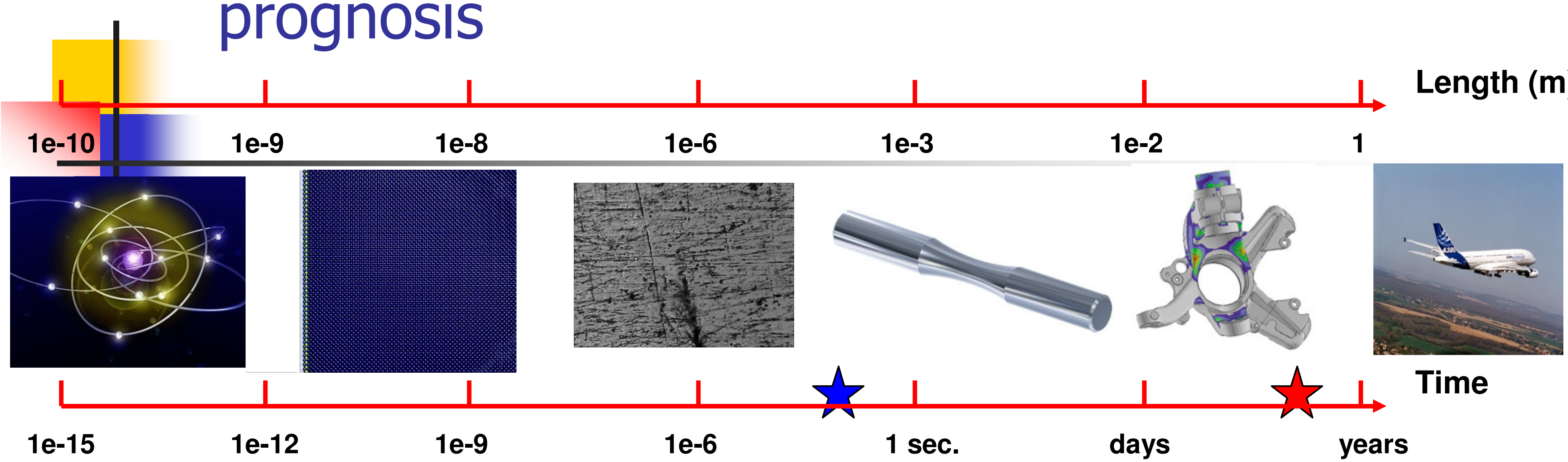
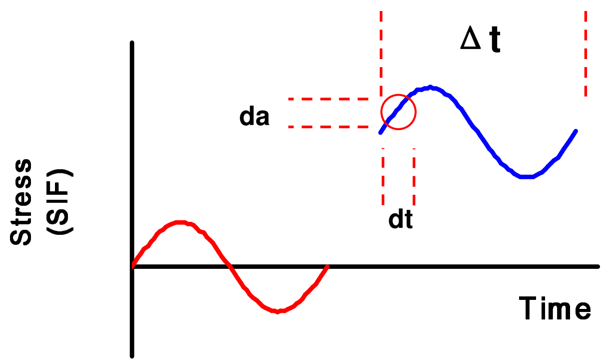
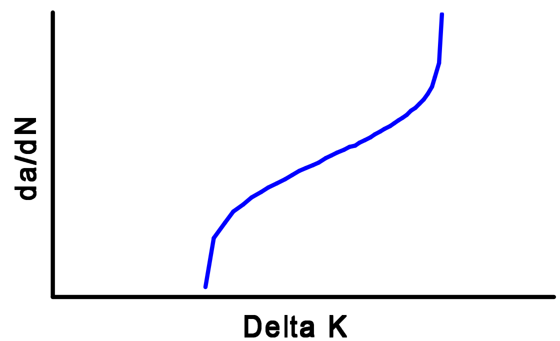
Multi-scale model for fatigue damage prognosis

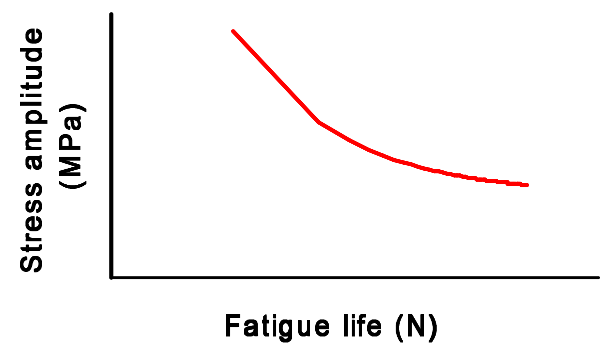




da/dt relationship at a smaller time scale



da/dN curve – Paris, 1960's



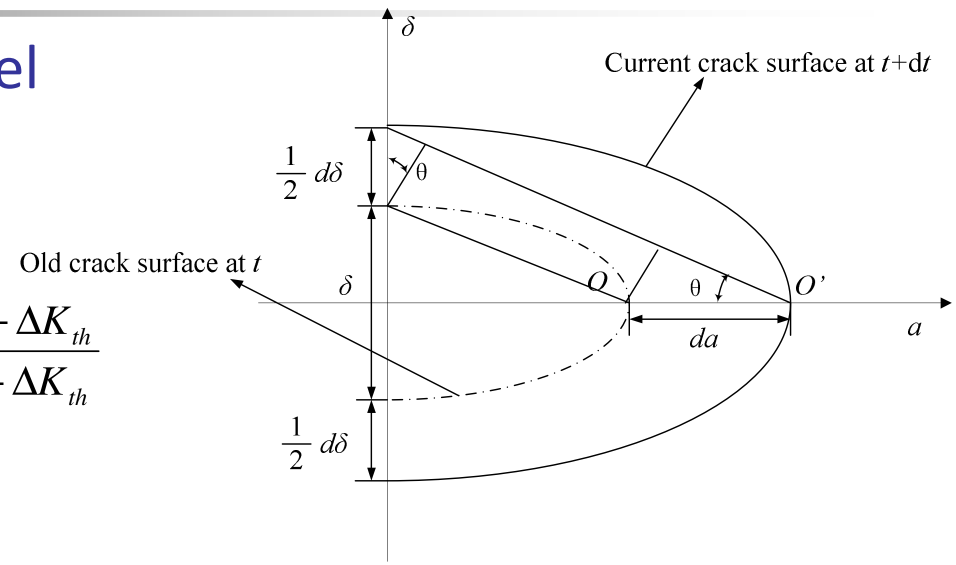
SN curve – Wholer, 1860's

Small time scale model [1]

Incremental crack growth model

$$\frac{1}{C\lambda a}\frac{da}{dt} = \frac{2\sigma}{1 - C\lambda\sigma^2}\frac{d\sigma}{dt}$$

where $\lambda = \frac{4}{E\sigma_y}$ $C = \frac{ctg\theta}{2}$ and $\theta = \frac{\pi}{2}\frac{\Delta K - \Delta K_{th}}{K_c - \Delta K_{th}}$

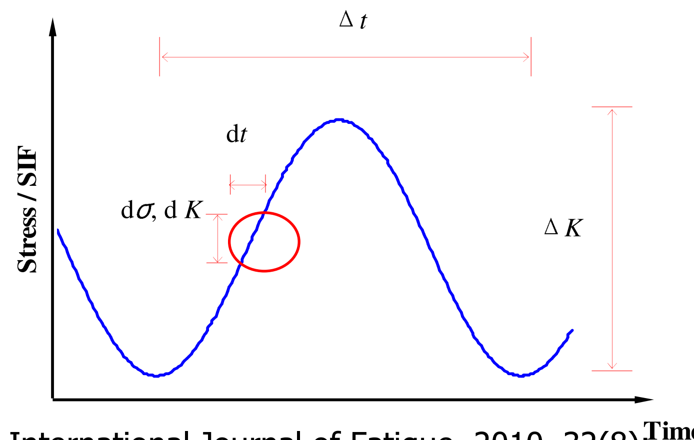


General crack growth kinetics

$$\dot{a} = H(\dot{\sigma}) \cdot H(\sigma - \sigma_{ref}) \cdot \frac{2C\lambda}{1 - C\lambda\sigma^2} \cdot \dot{\sigma} \cdot \sigma \cdot a$$

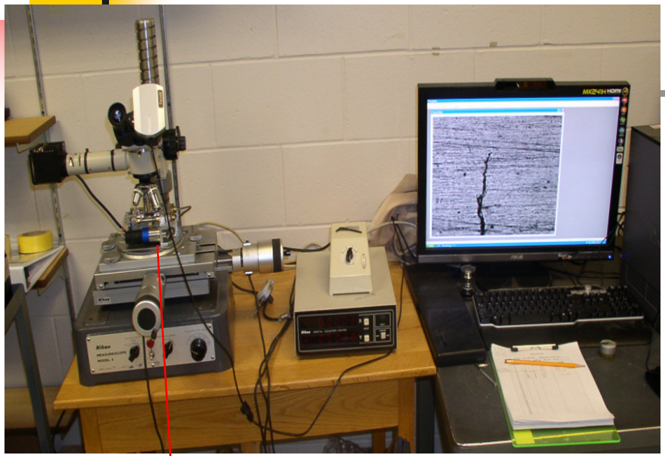
$$\sigma = f(t)$$

$$H(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x \le 0 \end{cases}$$

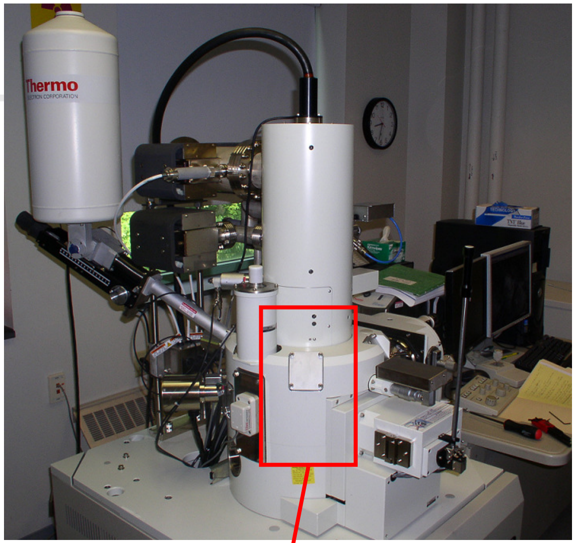


[1] Zizi Lu, Yongming Liu, Small time scale fatigue crack growth analysis. International Journal of Fatigue. 2010, 32(8): 1306-1321.

Hypothesis verification using in-situ fatigue testing

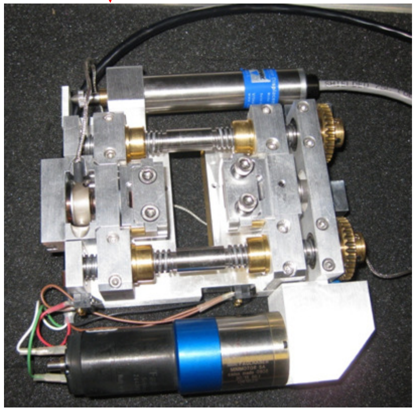


Nikon metallurgical microscope

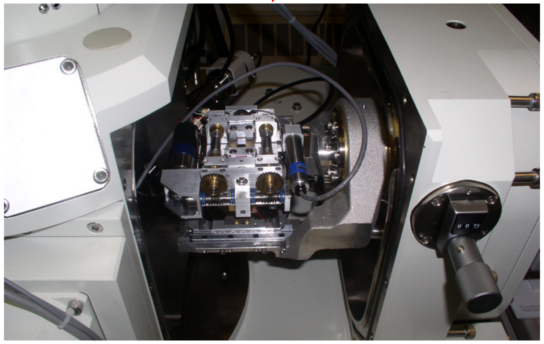


Jeol 7400-F SEM

Controller and PC



In-situ optical microscope fatigue testing

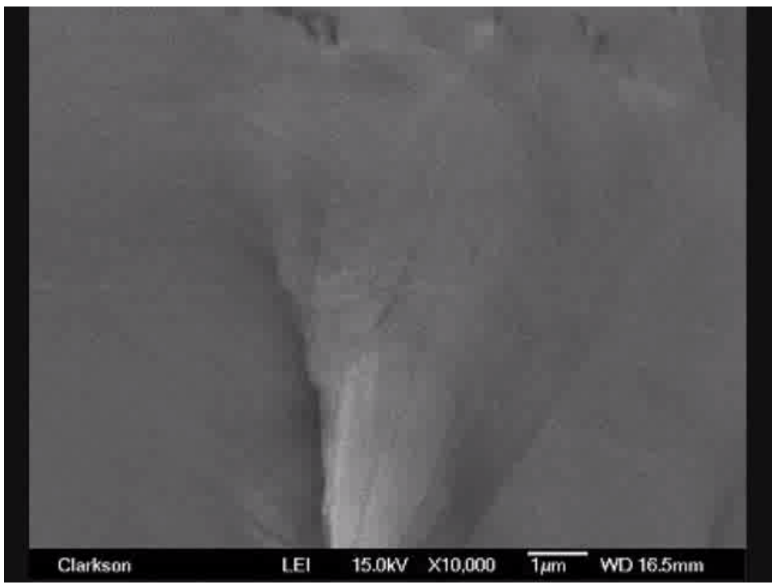


In-situ SEM fatigue testing

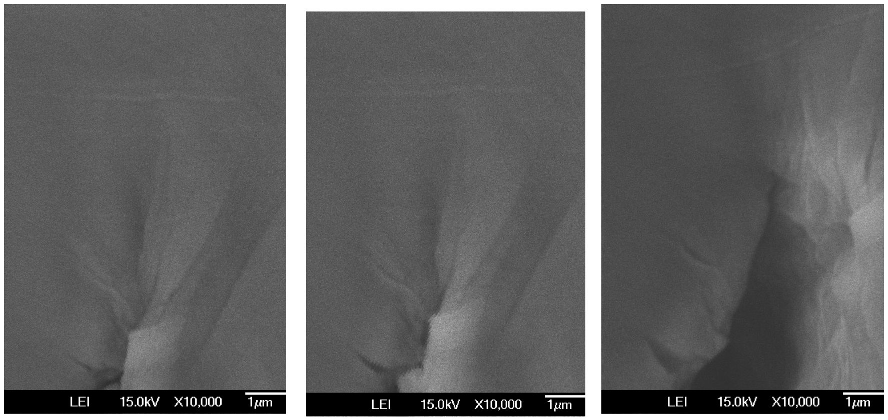
Crack closure observation

- Crack only starts to grow after a certain stress level (hypothesis 2)
- Crack closure is directly observed (hypothesis 1)
- Significant crack blunting is observed at the later stage of loading

Crack growth during loading



Loading increases



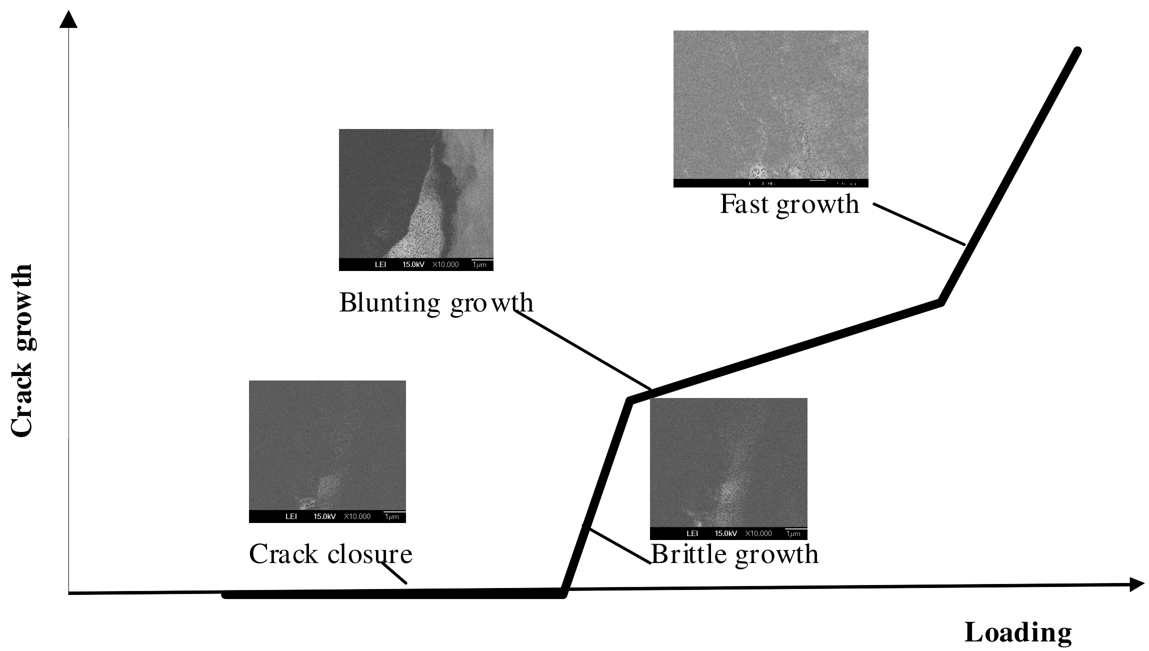
Fully close

Transition to open

Fully open

Multi-mechanism small time scale crack growth

- Multiple growth mechanisms within one cycle
- Cycle-based approaches focus on the average behavior per cycle and is not capable of describing this



Concurrent structural-material fatigue prognosis

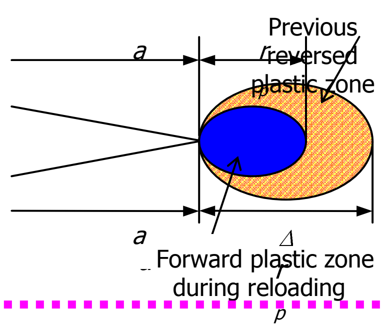
Structural dynamics

$$m\ddot{x}+n\dot{x}+kx=f(t)$$

Fatigue crack growth

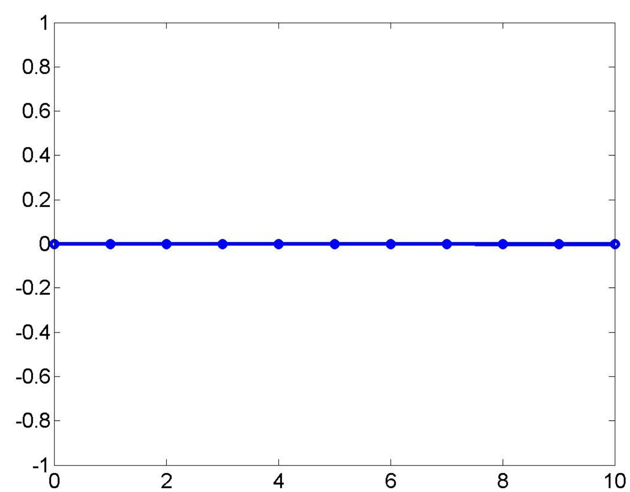
$$\dot{a}=H(\dot{\sigma})H(\sigma-\sigma_{ref})\frac{2C\lambda}{1-C\lambda\sigma^{2}}\dot{\sigma}\,a$$

$$H(x)=\begin{cases}1, & if\ \ x>0\\ 0, & if\ \ x\le 0\end{cases}$$



$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=(-k(x_3)/m)x_1+(-n/m)x_2+f(t)/m\\ \dot{x}_3=H(g(x_2))H(g(x_1)-\sigma_{ref})\dfrac{2C\lambda}{1-C\lambda g(x_1)^2}g(x_2)x_3\end{cases}$$

$$y=q(x_1,x_3)$$



Life prediction methodology

- Life prediction can be performed by time/cycle integration from the initial crack size to the final crack size
- Time-based approach

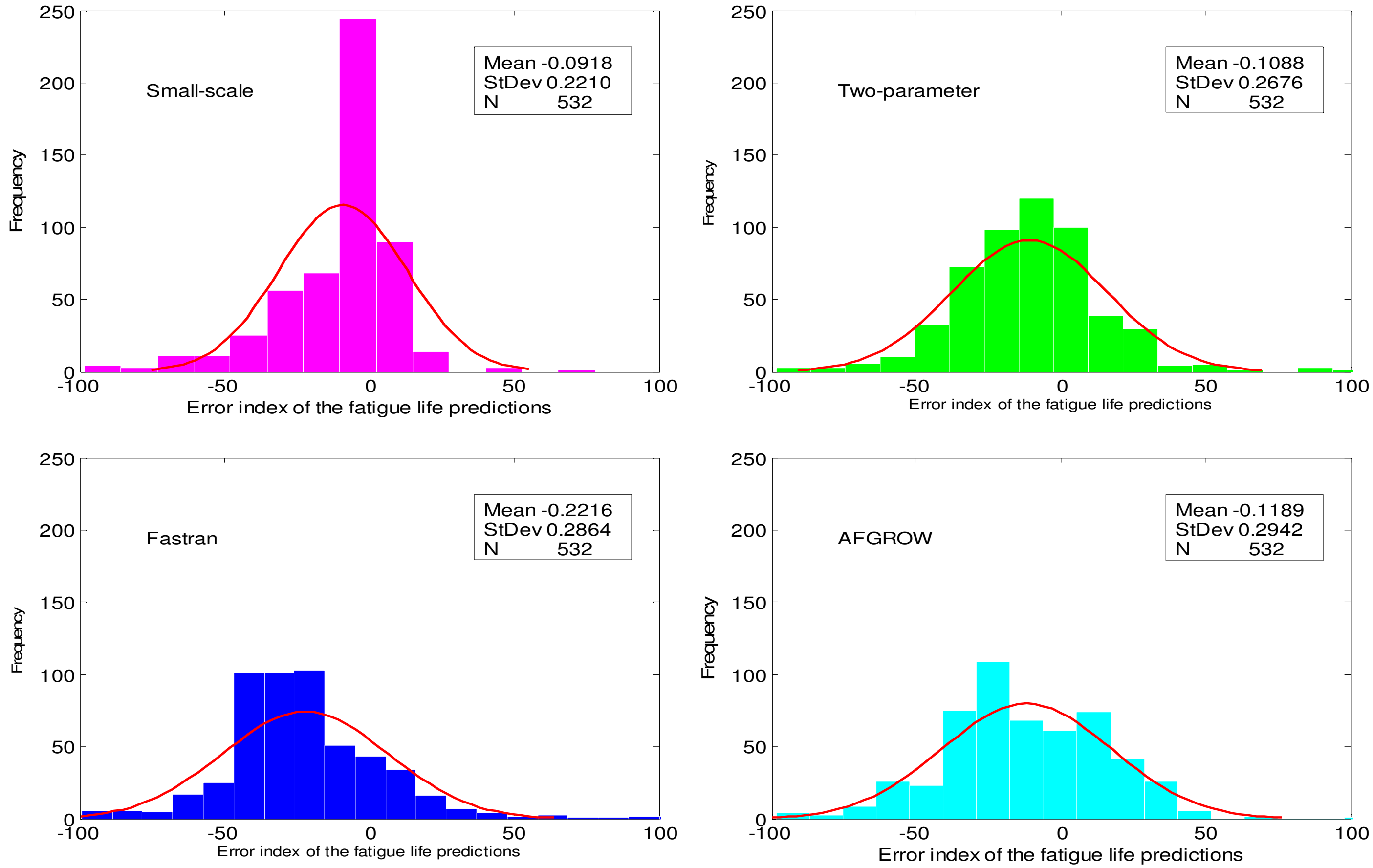
$$\dot{a} = H(\dot{\sigma}) \cdot H(\sigma - \sigma_{ref}) \cdot \frac{2C\lambda}{1 - C\lambda\sigma^2} \cdot \dot{\sigma} \cdot \sigma \cdot a \longrightarrow \int_{a_i}^{a_c} \frac{1}{a} da = \int_o^T H(f^{'}(t)) \cdot H(f(t) - \sigma_{ref}) \cdot \frac{2C\lambda}{1 - C\lambda f^2(t)} \cdot f^{'}(t) \cdot f(t) dt H$$

$$\sigma = f(t)$$

- Cycle-based approach

$$\frac{da}{dN} = f(\Delta K) \longrightarrow N = \int_0^N dN = \int_{a_i}^{a_c} g(\Delta K) da = \int_{a_i}^{a_c} \frac{1}{f(\Delta K)} da$$

Error analysis for experimental data under variable amplitude loadings



Benefits of the development of mechanism-based model

- Increase the mean/median prediction accuracy
- Reduce the model calibration requirements/uncertainties
- Enhance the predictability under unknown loading conditions
- Reduce modeling uncertainties by gaining knowledge about failure mechanisms

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Uncertainty quantification

Sources of uncertainties

- Physical variability

  - Material properties

  - Loading

  - Structural geometries

  - Initial conditions (damage size, residual stress, surface roughness, etc.)

- Information uncertainty

  - Insufficient data

  - Measurement noise

- Model uncertainty

  - Model form error

  - FEM mesh error

  - Surrogate modeling

  - Statistical distribution and their covariance structure

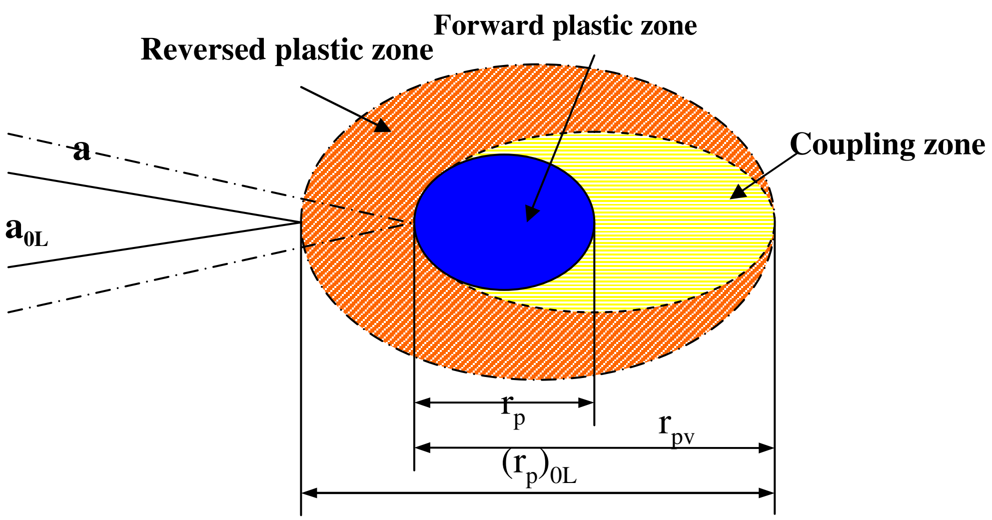
Surrogate modeling

- Surrogate model: approximate the complex mechanism model by response surface functions
- Different types of surrogate models
  - Least-square curve fitting
  - Gaussian Process regression
  - Polynomial chaos regression
  - Support vector machine
  - Relevance vector machine

Case study: retardation effect of fatigue crack growth

- Fatigue crack growth retardation happens when a large overload exists in the loading history
- Cycle-by-cycle calculation for the tracking of this behavior
- Surrogate model of a retardation factor for fast simulation
- Procedure

  - design of numerical experiments + STS simulation + response surface approximation



$$\Delta\sigma_{eq}^{\ *} = \eta\Delta\sigma_{eq}$$

$$\eta = k(R_{OL}, P_{OL})$$

R_{OL} : Overload ratio

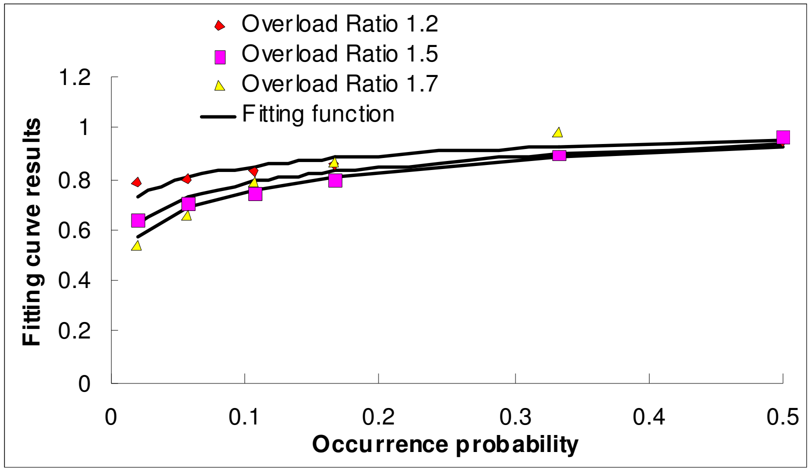
P_{OL} : Probability of overload cycles

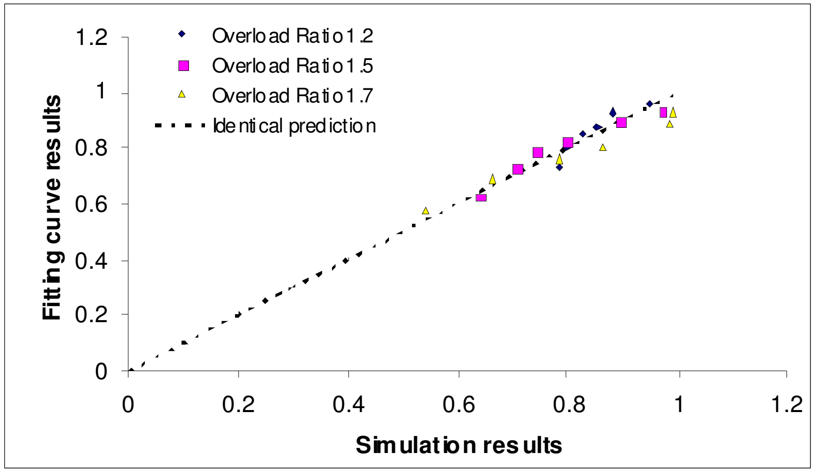
Least-square curve fitting

η : the coefficient for the load interaction effect

$$\eta = k(R_{OL}, P_{OL}) = 1 + A\log(P_{OL})(R_{OL} - 1)^{B}$$

$$A = 0.12, \qquad B = 0.38$$





Gaussian Process Model

Basic idea: model the output Y as a Gaussian process which is indexed by the inputs $\mathbf{x}$.

Training: Given m training points $\mathbf{x}_1,\ldots,\mathbf{x}_m$, with corresponding outputs $\mathbf{Y}=[Y(\mathbf{x}_1),\ldots,Y(\mathbf{x}_m)]^T$, the joint distribution of Y is defined by

$$\mathbf{Y} \sim N_m[\mathbf{f}^T(\mathbf{x})\boldsymbol{\beta}, \lambda K]$$

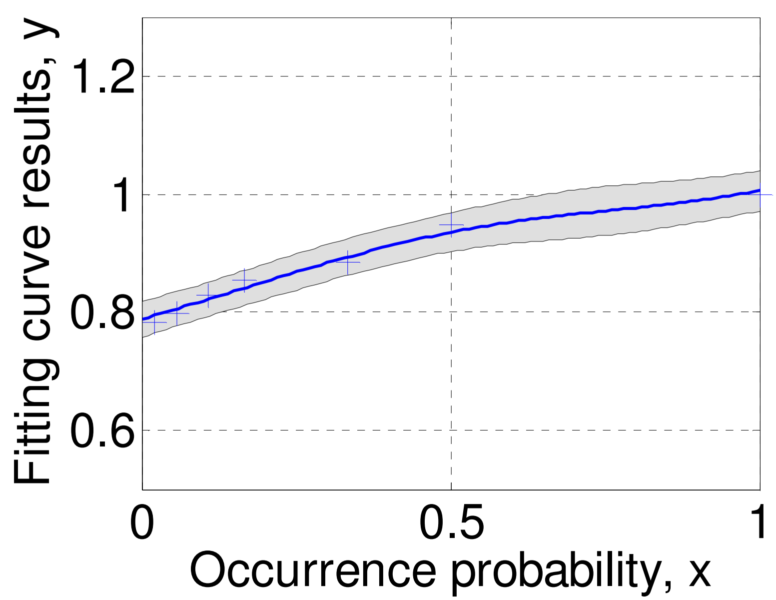
$\mathbf{f}^T$ – q basis functions for the trend -- linear or quadratic
β – coefficients of the regression trend
λ – process variance, $\lambda = \sigma^2$
K – m by m matrix of correlations among the training points

Predicting: At untested location $\boldsymbol{x}^*$, the prediction Y^* is given by

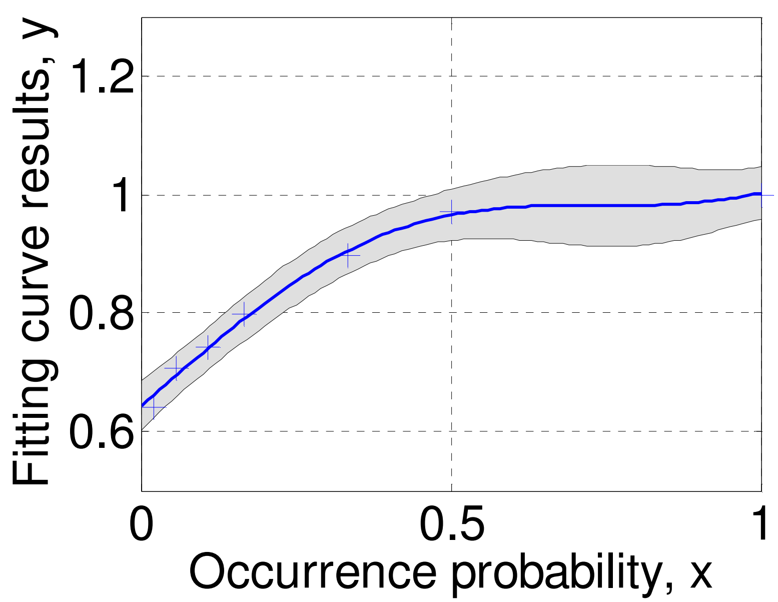
$$Y^* = \mathrm{E}\left[Y(\mathbf{x}^*) \mid \mathbf{Y}\right] = \mathbf{f}^T(\mathbf{x}^*)\boldsymbol{\beta} + r^T(\mathbf{x}^*)\mathbf{R}^{-1}(\mathbf{Y} - \mathbf{F}\boldsymbol{\beta})$$

$\mathbf{F}$ – m by q matrix, the trend basis functions at each of the training points;
r – the vector of correlations between $\mathbf{x}^*$ and each of the training points.

GP regression results

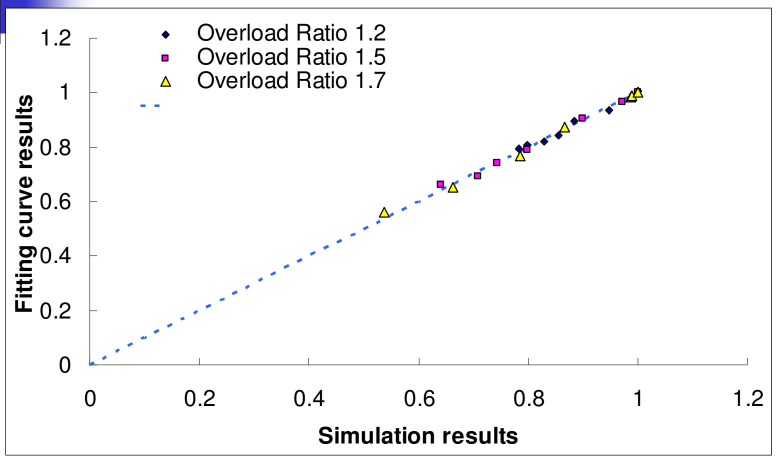


Overload ratio 1.2



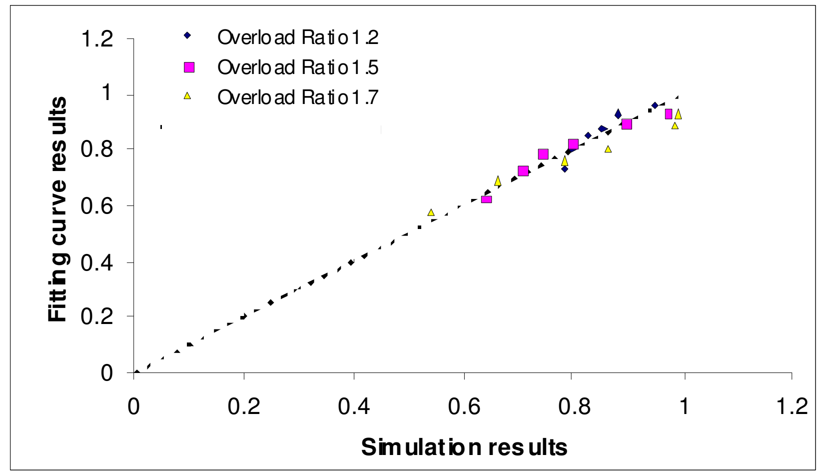
Overload ratio 1.5

Comparison between two surrogate models



GP regression

SSE	0.002653
Mean Error	1.25%
Max. Error	4.39%

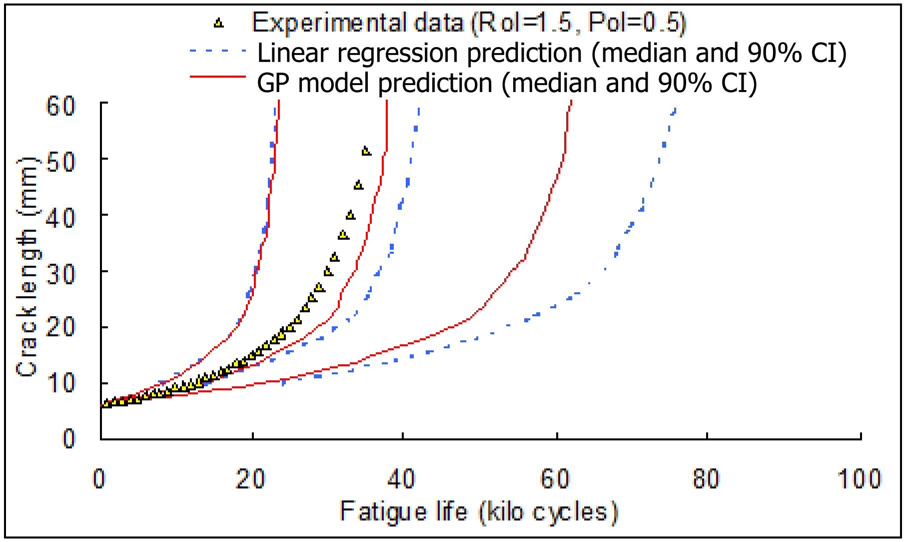


Least square regression

SSE	0.031709
Mean Error	3.57%
Max. Error	10.71%

- GP model improves the fitting accuracy and reduces the modeling error
- Small modeling error will propagate through fatigue model and will be amplified $\frac{1}{(1+\varepsilon)^m} - 1$

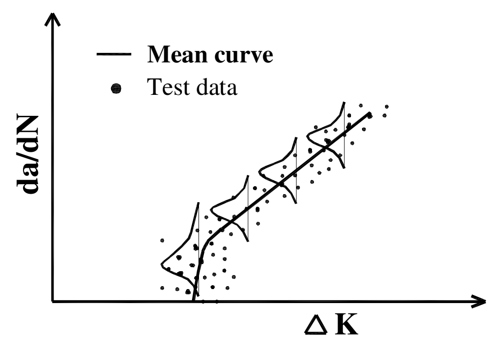
Impact on RUL prediction



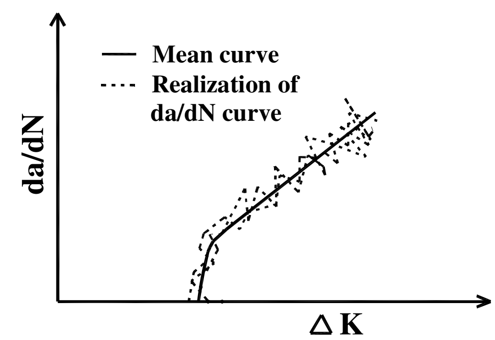
- Differences are observed for mean prediction and one confidence bound
- GP model has a smaller confidence bounds due to smaller modeling errors (24% reduction in confidence bounds)
- Modeling uncertainty is critical for some prognostics problems

Covariance structure of random variables

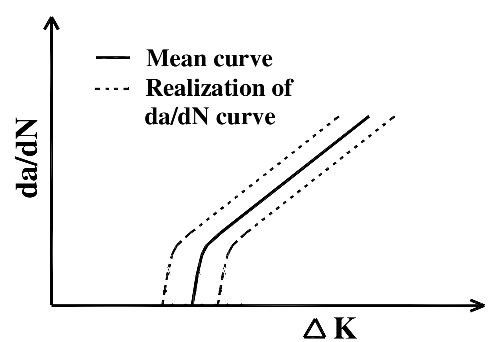
- Crack growth representation



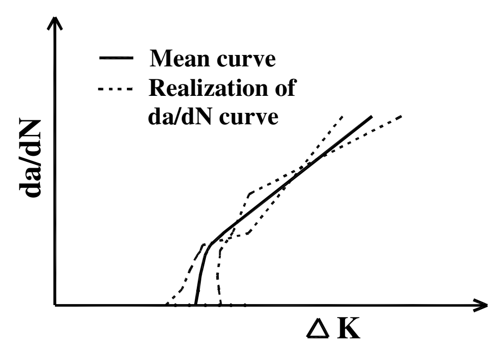
a) da/dN vs ΔK curve of experiments



b) white noise da/dN vs ΔK curve



c) percentile da/dN vs ΔK curve



b) stochastic da/dN vs ΔK curve

Importance of covariance structure

- Example of crack growth during two stress cycles

$$\Delta a_{total} = \Delta a_1 + \Delta a_2 = \left(\frac{da}{dN}\right)_{\Delta K_1} \times 1 + \left(\frac{da}{dN}\right)_{\Delta K_2} \times 1$$

$$mean(\Delta a_{total}) = mean(\Delta a_1) + mean(\Delta a_2)$$

$$Var(\Delta a_{total}) = Var(\Delta a_1) + Var(\Delta a_2) + 2\rho\sqrt{Var(\Delta a_1)Var(\Delta a_2)}$$

Covariance structure of crack growth curve will not affect deterministic prediction much but affect the uncertainty/reliability prediction

Clarkson UNIVERSITY defy convention

Random process representation of crack growth

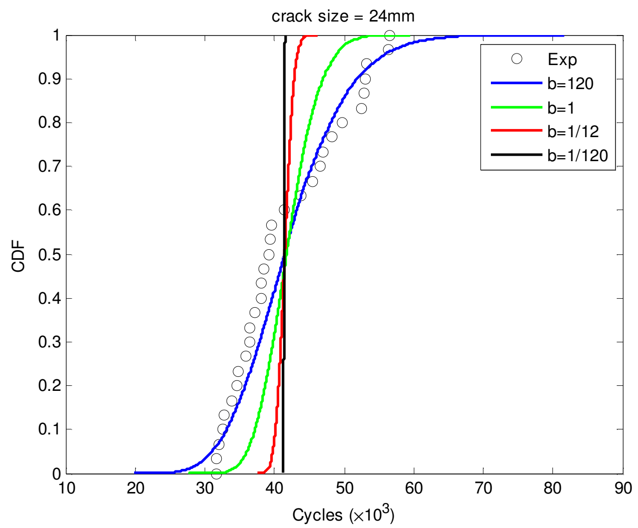
- Probabilistic crack growth prediction needs to consider the covariance/correlation among the basic input random variables
- Crack growth process is treated as a random process using Karhunen-Loeve random process expansion technique

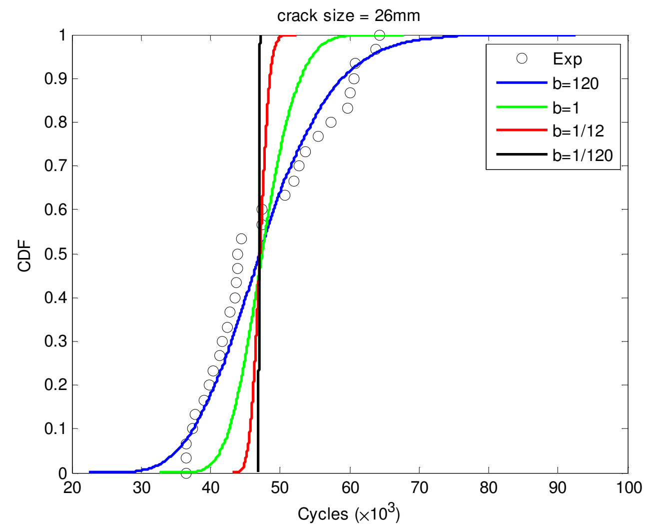
$$\underbrace{\varpi(x,\theta)}_{\text{Random process}} = \underbrace{\bar{\varpi}(x)}_{\text{mean}} + \sum_{i=1}^{\infty} \underbrace{\sqrt{\lambda_i}}_{\text{Eigenvalues}} \underbrace{\xi_i(\theta)}_{\text{Random variable}} \underbrace{f_i(x)}_{\text{eigenfunction}}$$

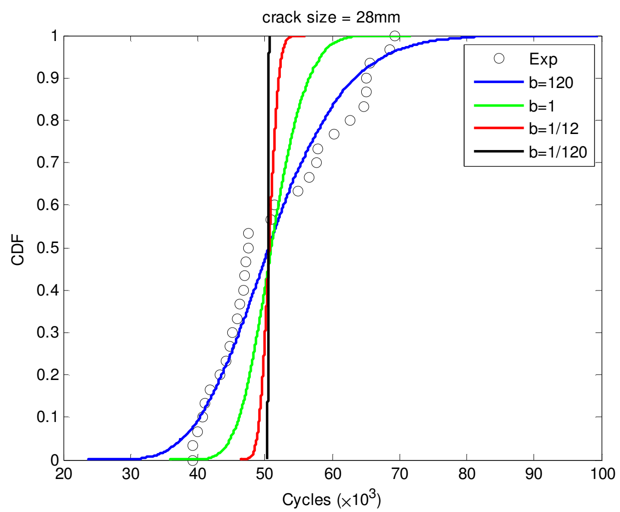
$$C(a_1,a_2) = \sigma e^{-\frac{|a_1-a_2|}{b}}$$ covariance function

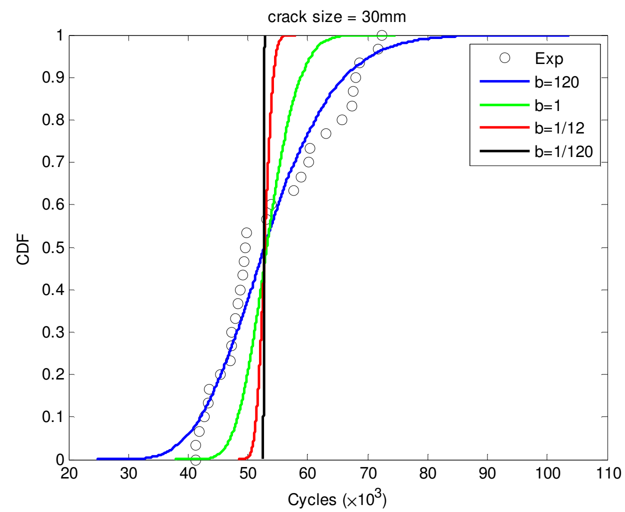
$b \to 0$ C=0, independent variables $b \to \infty$ C=1, fully correlated variables

Case study for Al-2024-T3







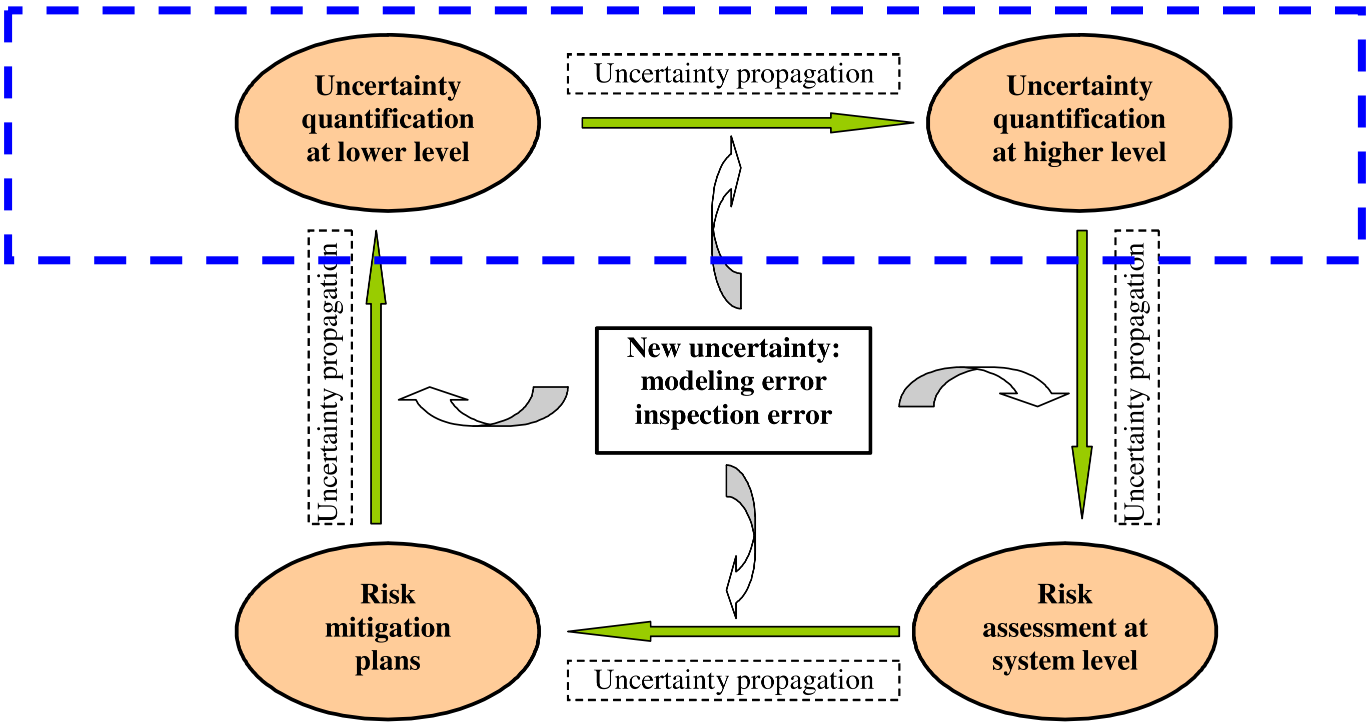


Summary

- Case study for fatigue crack growth analysis indicates a highly correlated crack growth process
- Percentile crack growth approach is a good approximation
- Other prognostics problems may have different covariance structures
- A flexible approach to handle the covariance structure of random variables

Uncertainty propagation -1

- Evaluate the uncertainty of response variables (RUL, crack size, stiffness degradation, etc.) by propagating quantified uncertainties through multiscale mechanism model



Uncertainty propagation -2

- Mathematically, uncertainty propagation evaluates the multidimensional integral in the random variable space

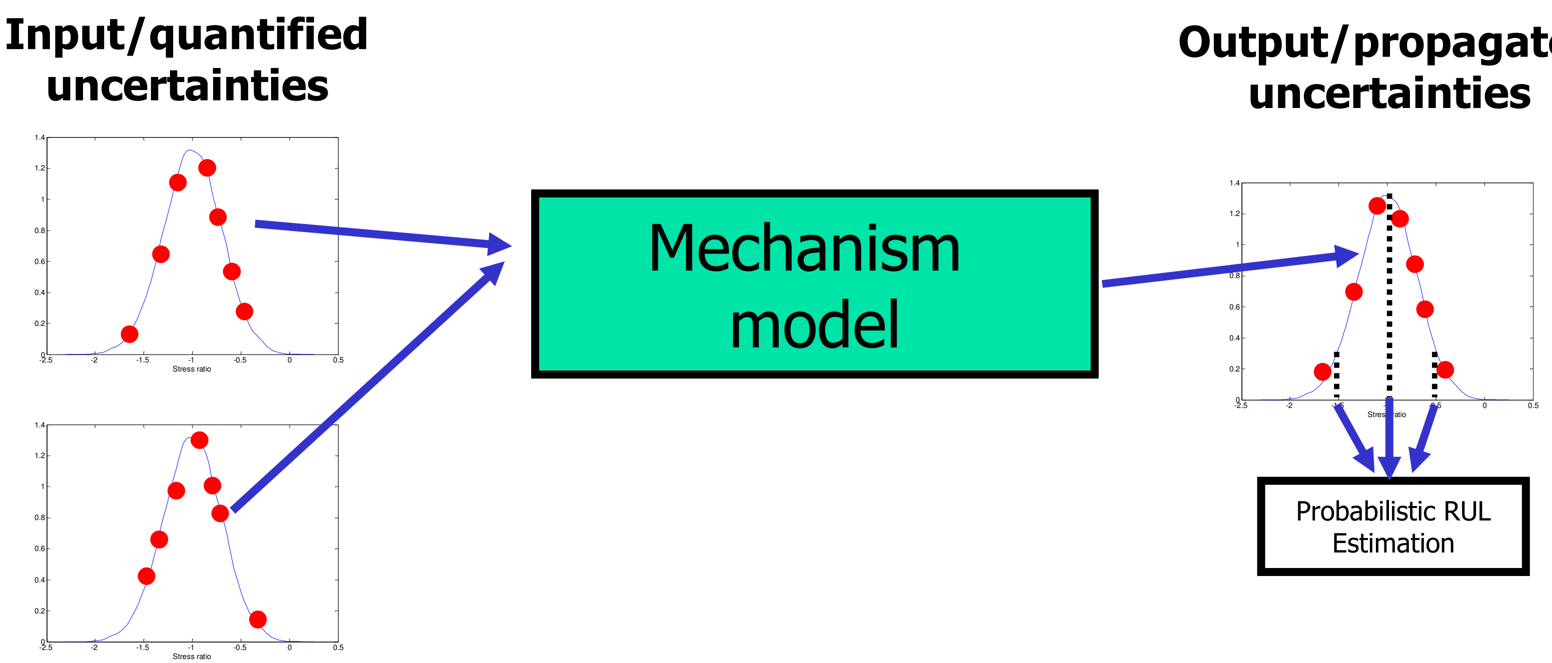
- Different approximation methods

  - simulation-based approach: direct Monte Carlo (MC) simulation; MC with importance sampling; particle filter, subset simulation

  - analytical approach: first-order reliability method (FORM); second-order reliability method (SORM)

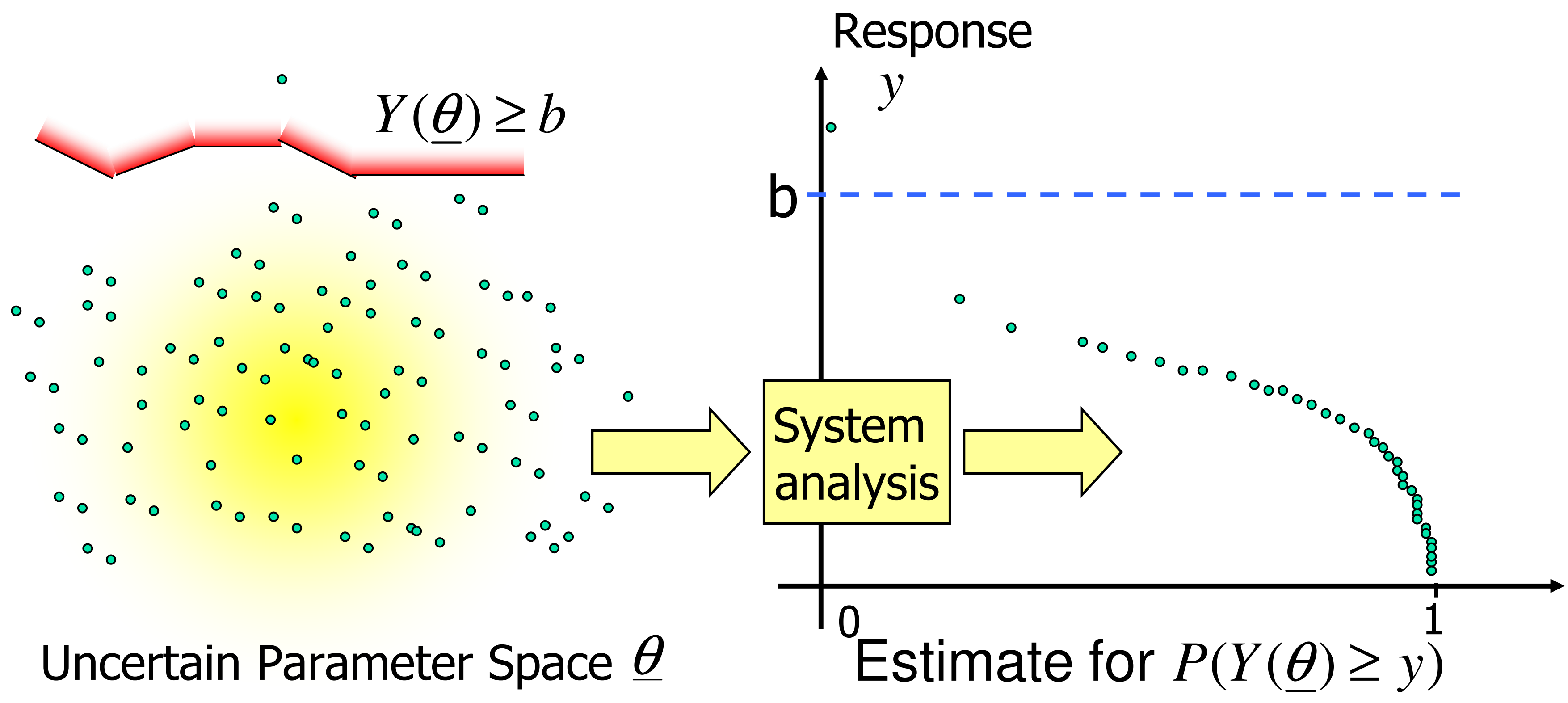
  - moment matching: univariate approximation; bivariate and multivariate approximation

Simulation-based approach for uncertainty propagation



- Key steps in simulation-based approach
  random number generator + model simulation + estimate probabilistic RUL
- Time consuming for complex models
- Samples are not efficient in direct MC simulation and sampling techniques can improve the computational efficiency

Sample efficiency of direct MC simulation



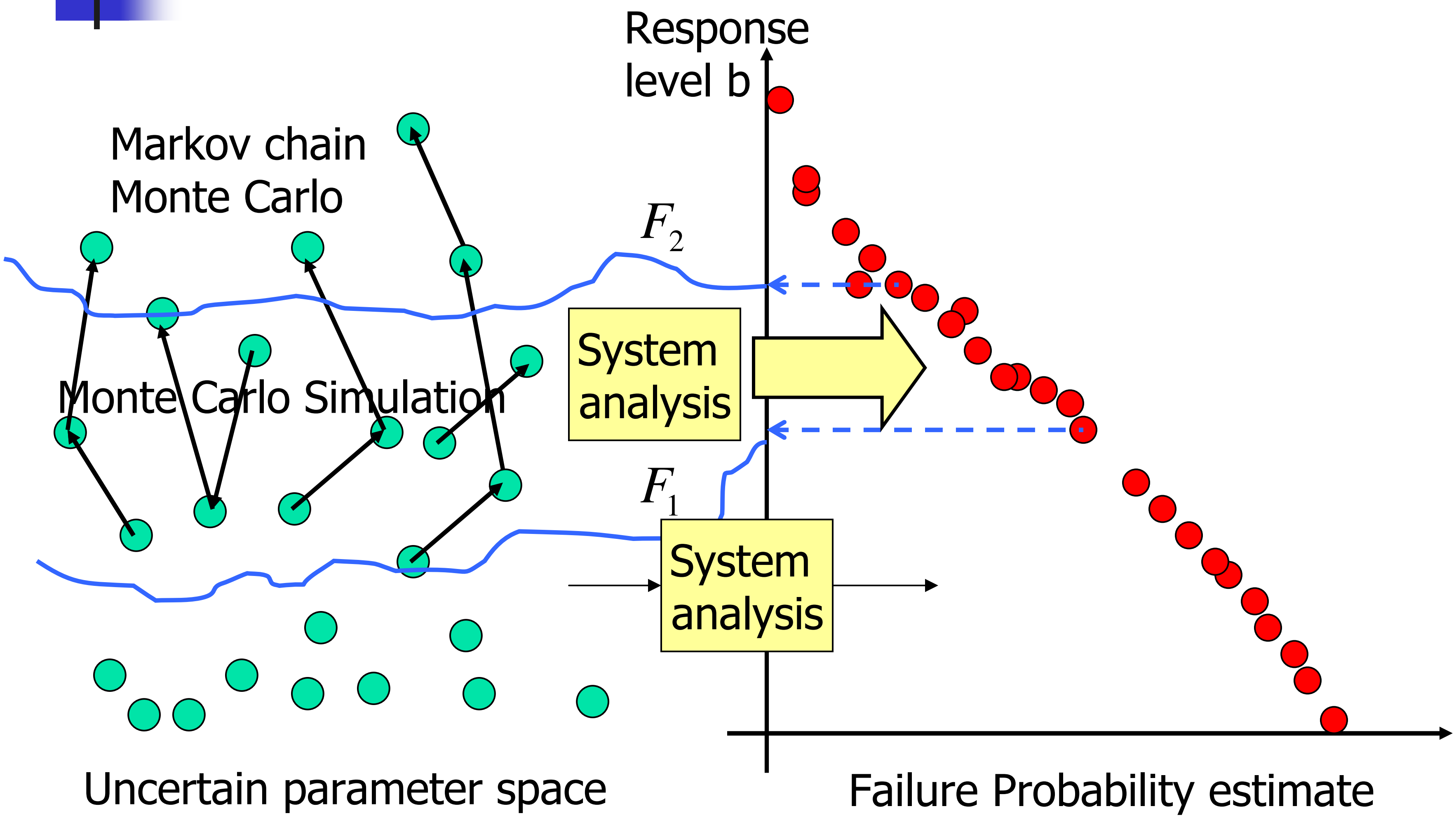
Subset Simulation - Basic Idea

- Express a small failure probability as a product of larger conditional failure probabilities

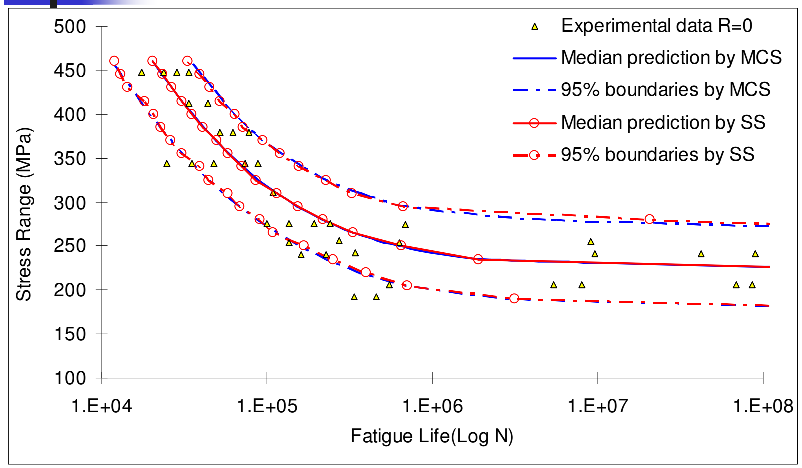
	$P(Y>3)$	$=$	$P(Y>3 \mid Y>2)$	$\times$	$P(Y>2 \mid Y>1)$	$\times$	$P(Y>1)$
	0.001	$=$	0.1	$\times$	0.1	$\times$	0.1
	rare		frequent		frequent		frequent
# samples:	10,000		???		???		???

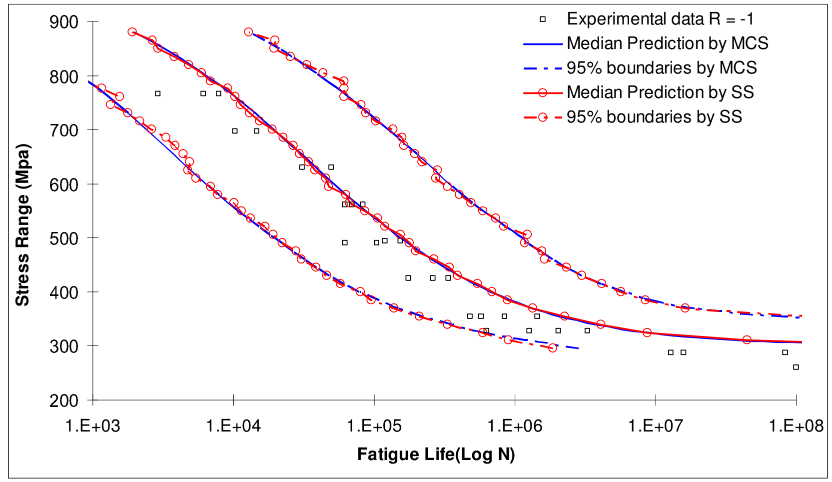
Require efficient simulation of conditional samples

Subset Simulation - Procedure



Case study: fatigue life prediction



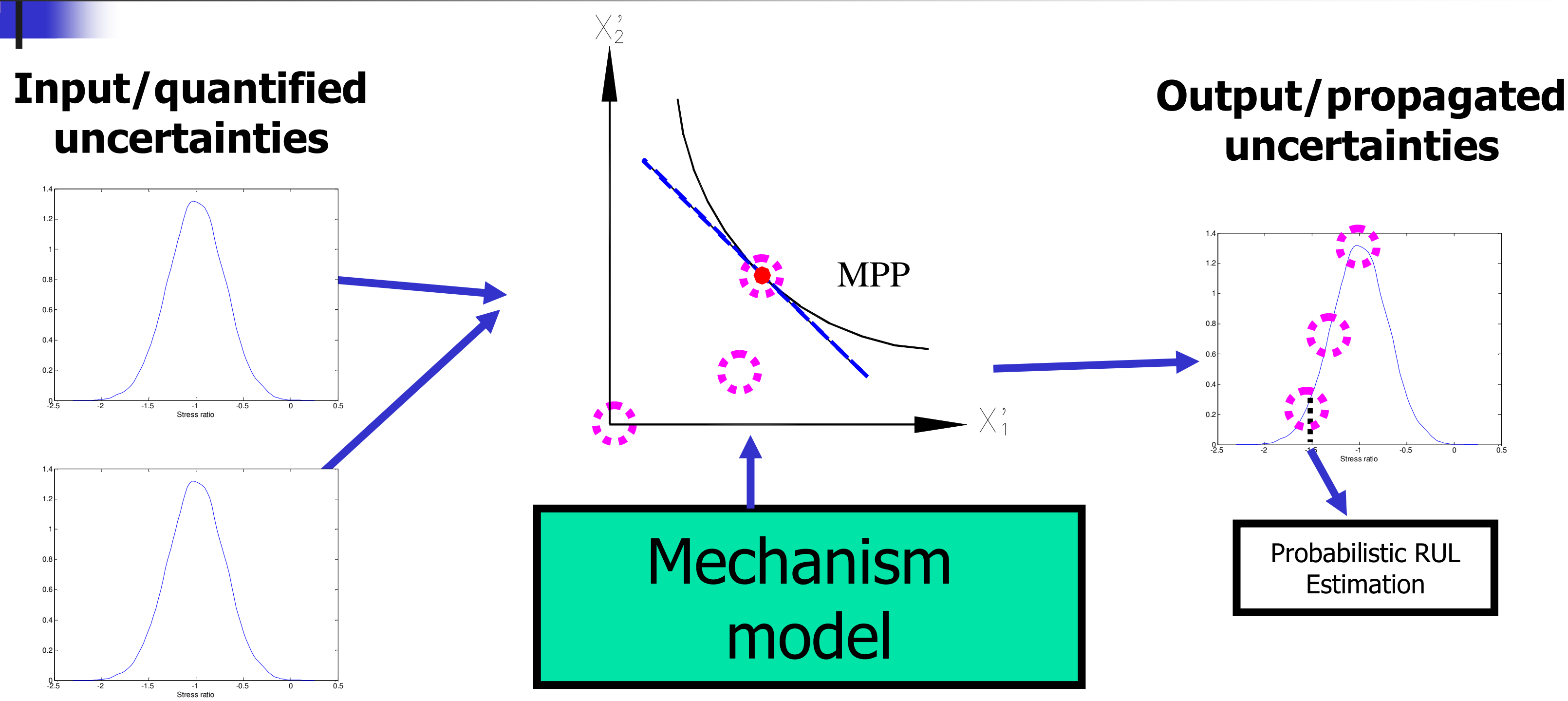


Al 7075-T6

Al 2024-T3

	Computational time	Average relative error	Maximum relative error
Direct MCS (one million samples)	~ 1.5 hour	-	-
Subset Simulation	~ 3 seconds	8.57 %	8.97 %

Analytical approach for uncertainty propagation



- Key steps in the analytical approach
  - Linear approximation of limit state function
  - Iteration to the convergence of a specified reliability/confidence
  - Estimate probabilistic RUL
- Applicable to linear/weakly nonlinear problems
- Transformation to standard normal space is required

Analytical approach - 1

- Fatigue reliability vs. probabilistic life prediction
  - Forward problem: reliability at a specified life
  - Inverse problem: life at a specified reliability level (prognosis)

- Governing equations and constraints

$$\begin{cases} (a): g(x,y)=0 \\ (b): \|x\| = \beta_{target} \\ (c): p_f = \Phi(-\beta_{target}) \\ (d): \begin{cases} (1) x + \dfrac{\|x\|}{\|\nabla_x g(x,y)\|} \nabla_x g(x,y) = 0 \quad (P_f < 50\%) \\ (2) x - \dfrac{\|x\|}{\|\nabla_x g(x,y)\|} \nabla_x g(x,y) = 0 \quad (P_f \geq 50\%) \end{cases} \end{cases}$$

x: vector of random variables (material properties, initial crack length, etc.)
y: vector of index variables (time and spatial coordinates)

[1] Xiang, Y. and Liu, Y., Application of Inverse first-order reliability method for probabilistic fatigue life prediction, Probabilistic Engineering Mechanics, 2010. (in revision)

Analytical approach - 2

First order Taylor's series expansion

- Considering x is varying and y is fixed

$$g(\mu, y) + (x - \mu)\cdot \nabla_x g(u, y) = 0$$

$$\begin{cases} x = \dfrac{[\nabla_x g(u,y)\cdot u] - g(u,y)}{\|\nabla_x g(u,y)\|^2}\nabla_x g(u,y) \\ y = y \end{cases}$$

$$d^1{}_k = \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \dfrac{[\nabla_x g(x,y)\bullet x] - g(x,y)}{\|\nabla_x g(x,y)\|^2}\nabla_x g(x,y) - x \\ 0 \end{bmatrix}$$

$$f^1(x,y) = \frac{1}{2}k_1\left\| x - \frac{[\nabla_x g(x,y)\bullet x] - g(x,y)}{\|\nabla_x g(x,y)\|^2}\nabla_x g(x,y)\right\|^2 + \frac{1}{2}k_2 g(x,y)^2$$

- Considering both x and y are varying

$$g(x,y) = g(\mu_x,\mu_y) + \nabla_x g(\mu_x,\mu_y)\cdot(x-\mu_x) + \nabla_y g(\mu_x,\mu_y)\cdot(y-\mu_y) + O(\mu_x,\mu_y) = 0$$

$$\begin{cases} x = -\beta_{target}\dfrac{\nabla_x g(x,y)}{\|\nabla_x g(x,y)\|} \\ y = \mu_y + \dfrac{[\nabla_x g(x,\alpha)\bullet x] - g(x,y) + \beta\,\|\nabla_x g(x,y)\|}{\dfrac{\partial g(x,y)}{\partial y}} \end{cases}$$

$$d^2{}_k = \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} -x - \beta_{target}\dfrac{\nabla_x g(x,y)}{\|\nabla_x g(x,y)\|} \\ \dfrac{[\nabla_x g(x,y)\bullet x] - g(x,y) + \beta_{target}\|\nabla_x g(x,y)\|}{\dfrac{\partial g(x,y)}{\partial y}} \end{bmatrix}$$

$$f^2(x,\alpha) = \frac{1}{2}k_3(\|x\| - \beta_{target})^2$$

Application to fatigue problems

- Limit state function

$$g() = \log\left(\int_{a_i}^{a_c} \frac{1}{AB^R[\Delta K - \Delta K_{th}]^m} da\right) - \log(N)$$

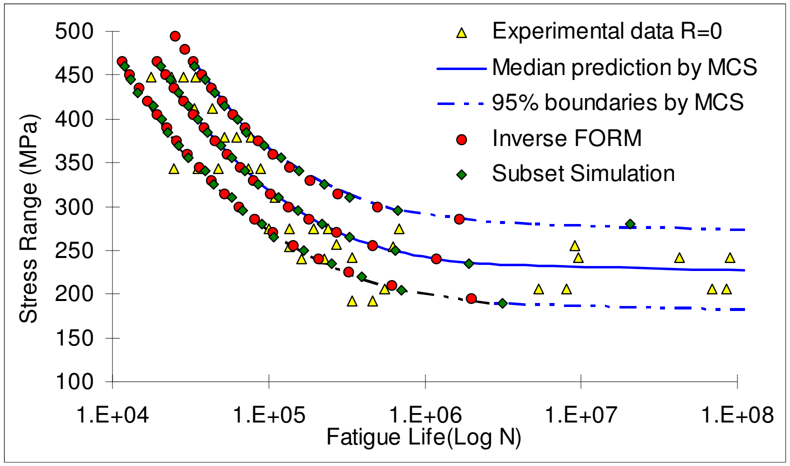
- Derivatives and searching algorithm in crack growth-based life prediction

$$\begin{cases} \nabla_x g(x,N) = \begin{cases} \dfrac{1}{A} \\ -\dfrac{\dfrac{1}{AB^R[\Delta K - \Delta K_{th}]^m}}{\displaystyle\int_{a_i}^{a_c} \dfrac{1}{AB^R[\Delta K - \Delta K_{th}]^m} da} \end{cases} \\ \dfrac{\partial g(x,N)}{\partial N} = -\dfrac{1}{N} \end{cases}$$

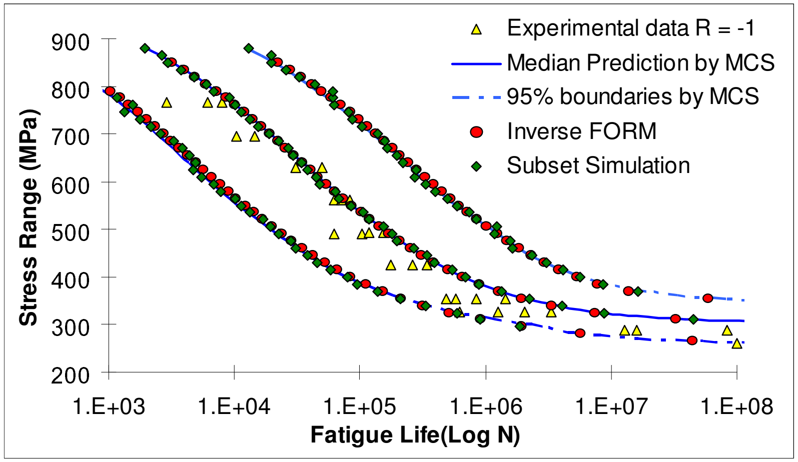
$$\begin{cases} X_{k+1} \\ N_{k+1} \end{cases} = \begin{cases} X_k + a_1\left(\dfrac{[\nabla_x g(x,N)\bullet x] - g(x,N)}{\|\nabla_x g(x,y)\|^2}\nabla_x g(x,N) - X_k\right) + a_2\left(-X_k - \beta_{t\arg et}\dfrac{\nabla_x g(x,N)}{\|\nabla_x g(x,N)\|}\right) \\ N_k + a_2\dfrac{[\nabla_x g(x,N)\bullet x] - g(x,N) + \beta_{t\arg et}\|\nabla_x g(x,N)\|}{\dfrac{\partial g(x,N)}{\partial N}} \end{cases}$$

Clarkson UNIVERSITY defy convention

Case study: life prediction



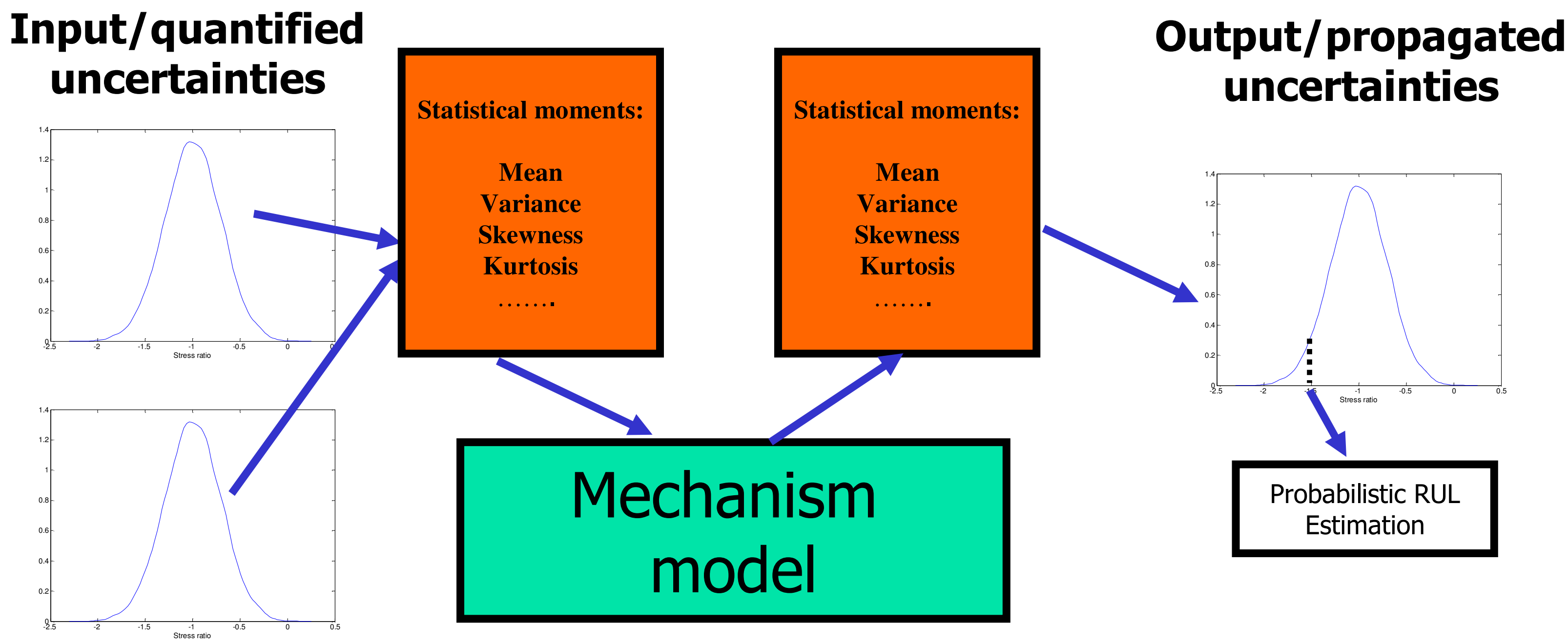
Al 7075-T6



Al 2024-T3

	Computational effort	Average relative error	Maximum relative error
Direct MCS (one million samples)	~ 1.5 hour	-	-
Inverse FORM	< 1 second	5.06 %	9.18 %
Subset Simulation	~ 3 seconds	8.57 %	8.97 %

Moment matching approach for uncertainty propagation



- Key steps in the moment matching approach
  - Calculate the statistical moments of input/quantified random variables
  - Estimate the statistical moments of output random variables
  - Approximate probability density function of output variables using moment information
  - Estimate probabilistic RUL
- Gradient and transformation free
- Most suitable for problems with less coupling among input variables

Limit state function approximation

- Decomposition of limit state function

$$g(\mathbf{x}) \approx \hat{g}_s(\mathbf{x}) = g_{\boldsymbol{\mu}} + \sum_{i=1}^{n} g_i(x_i) + \sum_{i_1,i_2=1;i_1<i_2}^{n} g_{i_1 i_2}(x_{i_1}, x_{i_2}) + \cdots + \sum_{i_1,\ldots,i_s=1;i_1<\ldots<i_s}^{n} g_{i_1 \cdots i_s}(x_{i_1}, \ldots, x_{i_s})$$

Mean value

Univariate function

Bivariate function

Multivariate function

$$g_{\boldsymbol{\mu}} = g(\mu_1, \mu_2, \ldots, \mu_n)$$

$$g_i(x_i) = g(\mu_1, \mu_2, \ldots, \mu_{i-1}, x_i, \mu_{i+1}, \ldots, \mu_n)$$

$$g_{i_1 i_2}(x_{i_1}, x_{i_2}) = g(\mu_1, \ldots, \mu_{i_1-1}, x_{i_1}, \mu_{i_1+1}, \ldots, \mu_{i_2-1}, x_{i_2}, \mu_{i_2+1}, \ldots, \mu_n)$$

- Truncated terms can be used to save the computational cost

  - no strong coupling among multiple terms

Moment estimation - 1

- Raw moment of response variable

$$m_l = E\left[g^l(\mathbf{x})\right] \approx E\left[\hat{g}_s^l(\mathbf{x})\right] = \sum_{i=0}^{s}(-1)^i \binom{n-s+i-1}{i} \times \sum_{i_1,\ldots,i_{s-i}=1;i_1<\cdots<i_{s-i}}^{n} E\left[\hat{g}_i^l\left(x_{i_1},\ldots,x_{i_{s-i}}\right)\right] \longrightarrow$$

$$\mu_g = m_1$$

$$\sigma_g = \sqrt{m_2 - m_1^2}$$

$$\alpha_{3g} = \frac{m_3 - 3m_1m_2 + 2m_1^3}{\sigma_g^3}$$

$$\alpha_{4g} = \frac{m_4 - 4m_1m_3 + 6m_1^2m_2 - 3m_1^4}{\sigma_g^4}$$

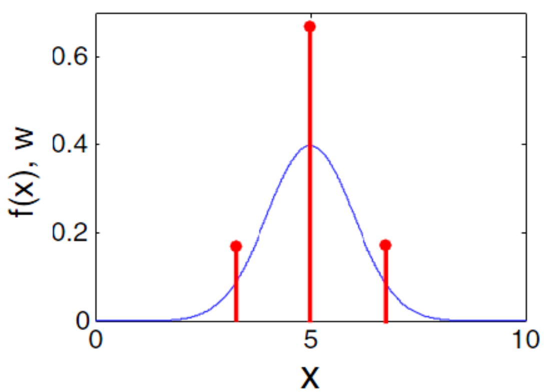
- Integration is required to calculate the expected value of $g^l(x)$

$$\mu'_n = \mathrm{E}(X^n) = \int_{-\infty}^{\infty} x^n \, dF(x)$$

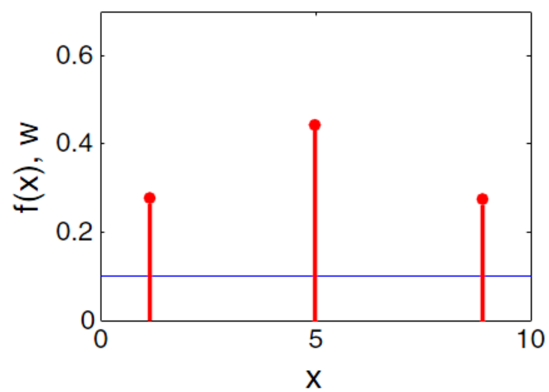
Moment estimation - 2

- Gauss-type quadrature is used for fast estimation of integrations

$$m_l = \int g(x)^l f_X(x)dx \approx \sum_{i=1}^{m} w_i g^l(x_i)$$



normal distribution $\mu = 5, \sigma = 1$



uniform distribution $0 \le x \le 10$

Approximation of probability density function

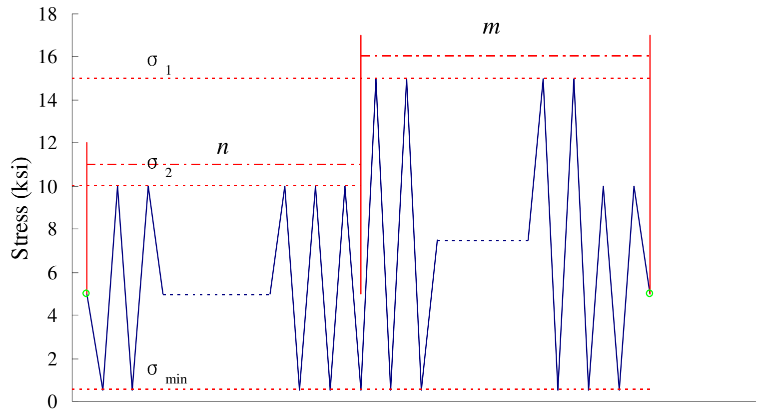
- Fitting classical distribution functions from calculated statistical moments in a least square sense
- Use a series expansion of Gaussian distribution to approximate the target cumulative distribution function (Edgeworth expansion)

$$F(g) \approx \Phi\left(\frac{g-\mu_g}{\sigma_g}\right) - \frac{\alpha_{3g}}{3!}\Phi^{(3)}\left(\frac{g-\mu_g}{\sigma_g}\right) + \frac{\alpha_{4g}-3}{4!}\Phi^{(4)}\left(\frac{g-\mu_g}{\sigma_g}\right)$$
$$+\frac{10\alpha_{3g}^2}{6!}\Phi^{(6)}\left(\frac{g-\mu_g}{\sigma_g}\right)+\cdots$$

$\Phi(j)(x)$ is the j-th derivative of $\Phi(\cdot)$ at point x

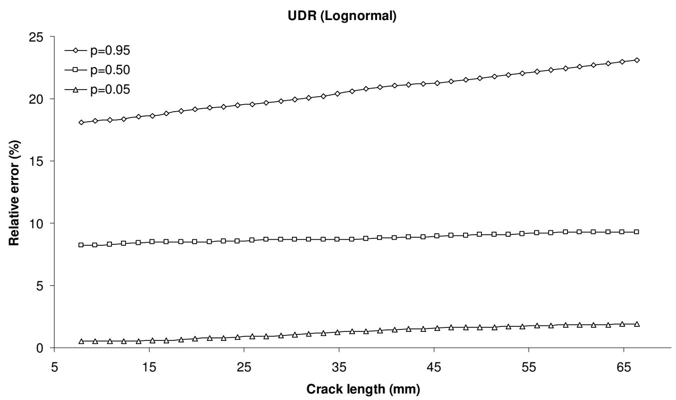
Case study: fatigue crack growth predictions

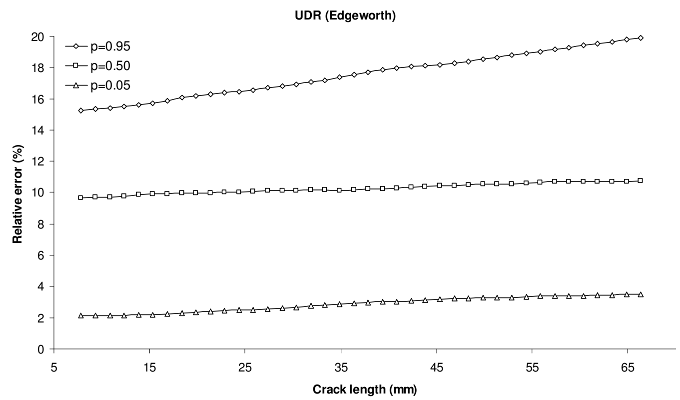
- Eight sets of experimental data under different spectrum loading (Al 7075-T6)
- Direct MC simulation, univariate approximation, and bivariate approximation are used for prediction
- Lognormal fitting and Edgeworth expansion are used to estimate crack length CDF

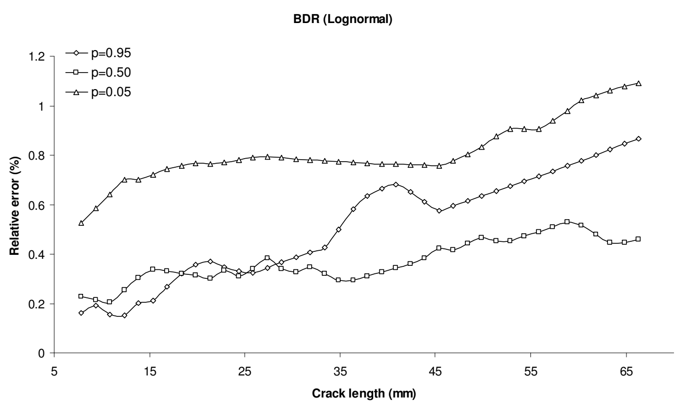


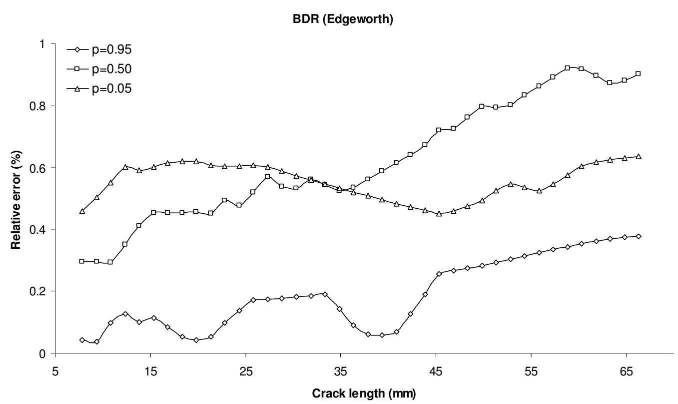
Schematic illustration of the two blocks loading

Parametric study for relative error analysis



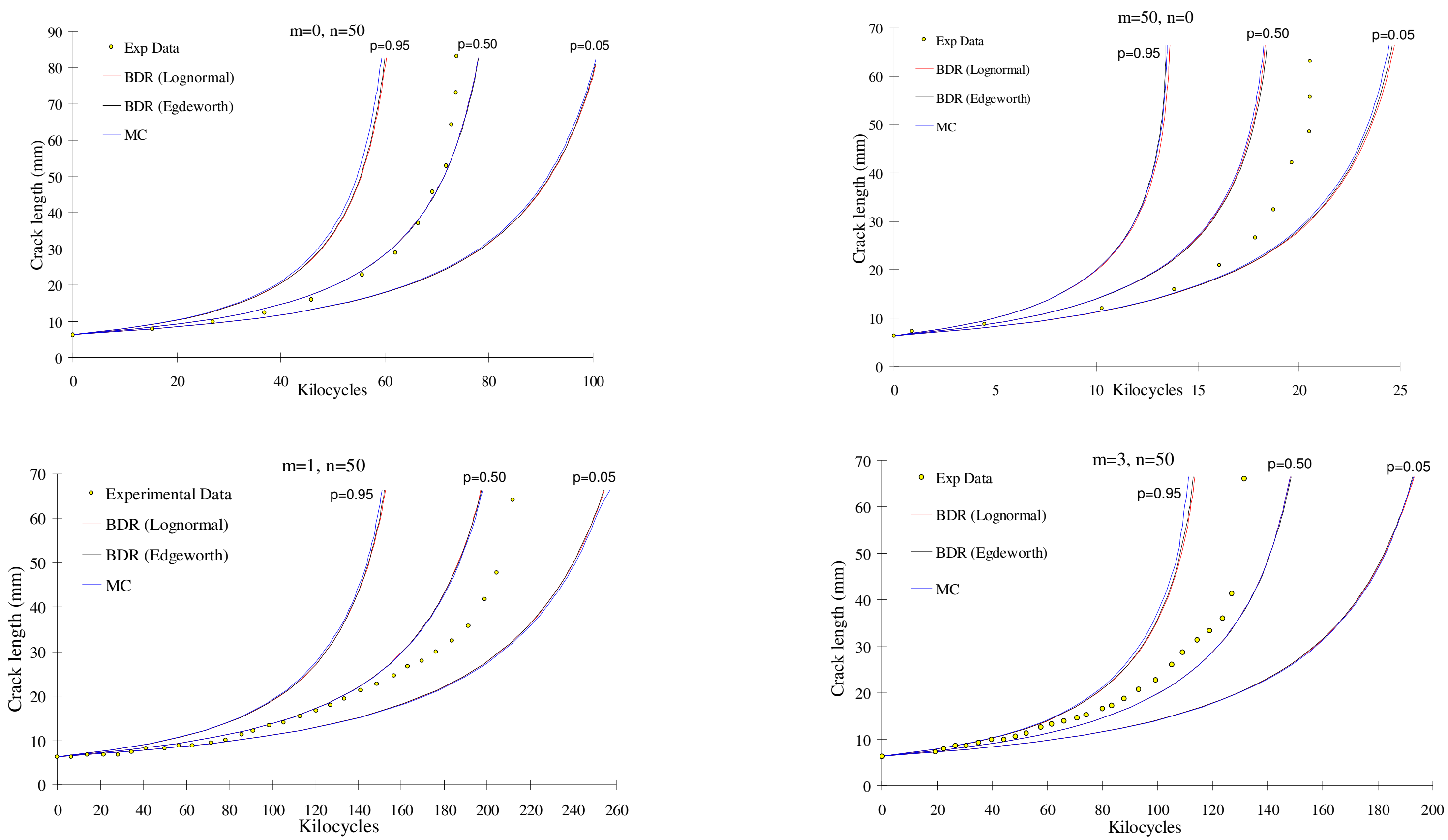




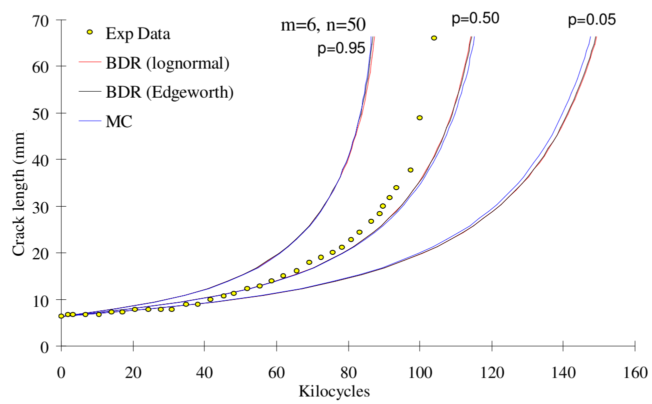


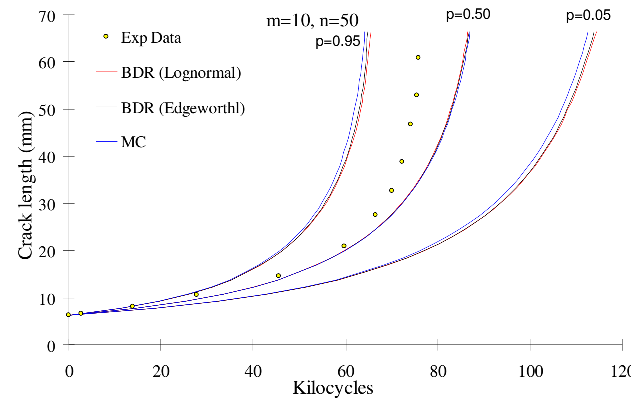
UDR method has large error and smallest computation time
BDR method has the best balance of computational efficiency and accuracy
Computational time for BDR is two magnitudes less than that of direct MC

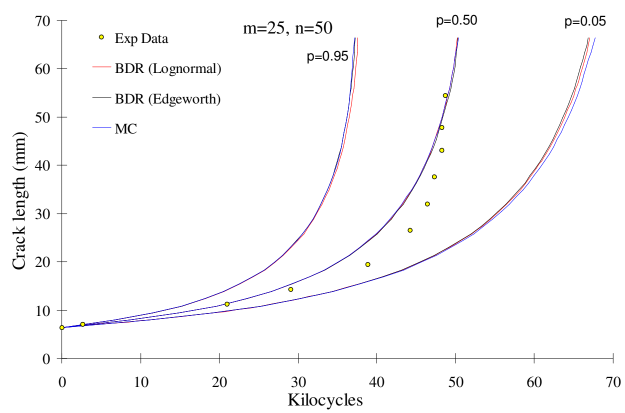
Model predictions vs. experimental data -1

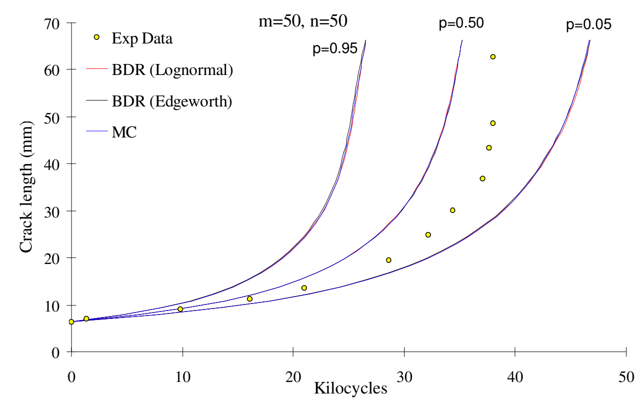


Model predictions vs. experimental data -2









Summary

- Three different uncertainty propagation methodologies
- All of them are very efficient compared to direct MC method
- Great potential for real-time prognosis of large systems
- Applicability of different methods depends on the specific problem

Outline

- Background and introduction
- Uncertainty management in prognostics
  - Model-based fatigue damage prognosis
  - Uncertainty quantification and propagation
  - Information fusion and uncertainty updating
  - Model comparison and validation
- Conclusions

Information fusion

- Above discussion focuses on the prognosis with “existing knowledge”
- “New information” from health and usage monitoring system and nondestructive inspection can help the prognostics
- Question? – how to fuse “existing knowledge” and “new information” for the model-based prognostics?

Bayesian updating

- Bayesian updating: update the belief of “existing knowledge” given “new information”

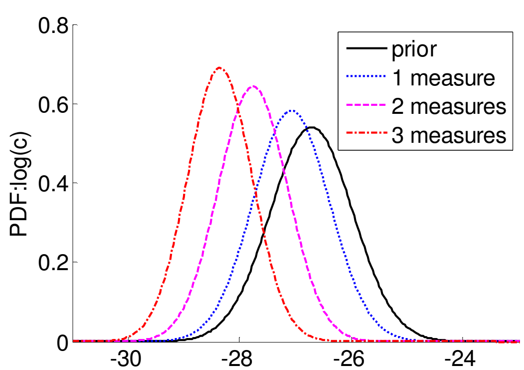
Posterior
New knowledge

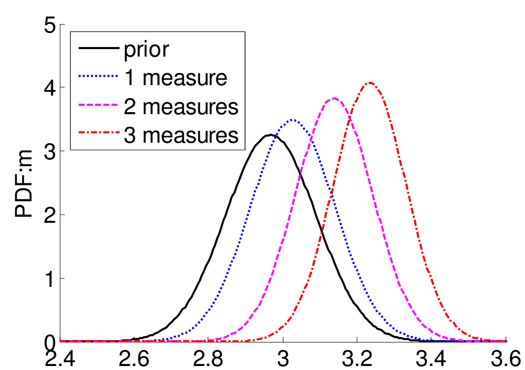
Likelihood
Model

Prior
Existing knowledge

$$P(\theta \mid x') = \frac{P(x' \mid \theta)P(\theta)}{P(x')}$$

Normalizing constant





Thomas Bayes

Maximum relative entropy (MRE) updating

- Relative entropy

$$S[P, P_{old}] = -\int dx d\theta P(x,\theta) \log \frac{p(x,\theta)}{p_{old}(x,\theta)} \qquad (1)$$

- Prior:

$$P_{old} = P_{old}(\theta) P_{old}(x|\theta) \qquad (2)$$

- Constraints: Data and Moment,

$$\int dx d\theta P(x,\theta) = 1 \qquad (3)$$

$$C_1 \ : \ P(x) = \int d\theta P(x,\theta) = \delta(x - x') \qquad (4)$$

$$C_2 \ : \ \int dx d\theta P(x,\theta) f(\theta) = \langle f(\theta) \rangle = F \qquad (5)$$

[1] A. Giffin and A. Caticha (2007). Updating probabilities with data and moments. In K. Knuth (Ed.), Bayesian Inference and Maximum Entropy Methods in Science and Engineering, AIP Conference Proceedings, 954:74

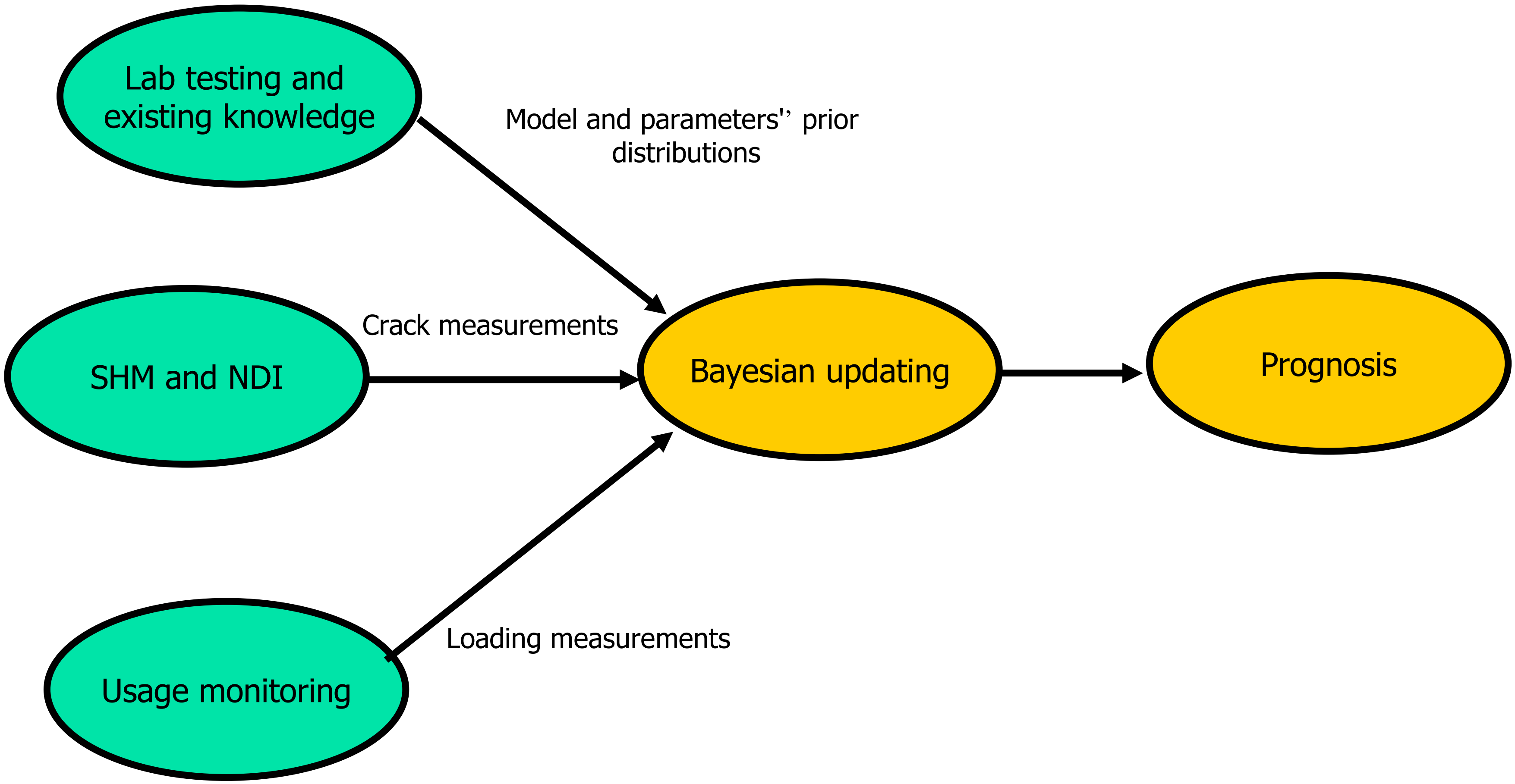
Comparison of Bayesian and MRE updating

- Maximum relative entropy (MRE) updating provides a generalized framework [1-2]
  - Classical Bayesian updating $\boxed{p(\theta) \propto \mu(\theta) \cdot \mu(x' \mid \theta)}$
  - Generalized Bayesian updating $p(\theta) \propto \mu(\theta) \cdot \mu(x' \mid \theta) \cdot \boxed{e^{\beta \cdot g(\theta)}}$
- Classical Bayesian updating handles point observations
- MRE updating can handle both point observations and other types of information
  - population information in terms of moments
  - interval information comes from physical limits and experts' opinion

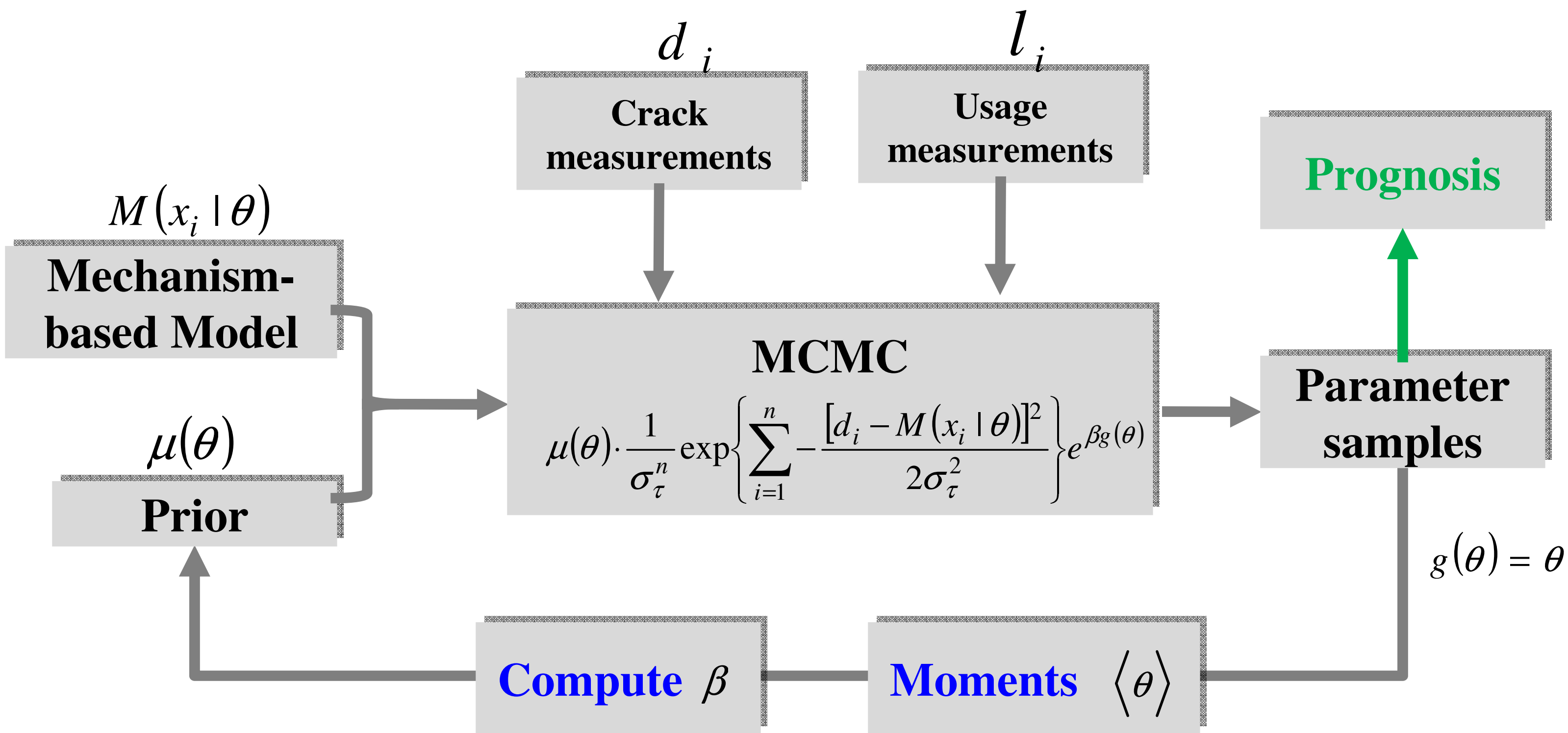
[1] A. Giffin and A. Caticha (2007). Updating probabilities with data and moments. In K. Knuth (Ed.), Bayesian Inference and Maximum Entropy Methods in Science and Engineering, AIP Conference Proceedings, 954:74

[2] Guan, X., Jha, R., **Liu, Y.**, "Probabilistic fatigue damage prognosis using maximum entropy approach", Journal of Intelligent Manufacturing, 2009. (accepted)

Information fusion in fatigue damage prognostics



Flow chart of generalized Bayesian updating



[1] Guan, X., **Liu, Y.**, Saxena, A., Celaya, J., Goebel, K., "Entropy-based probabilistic fatigue damage prognosis and algorithmic performance comparison", annual conference of the prognostics and health management society, San Diego, CA, 2009.

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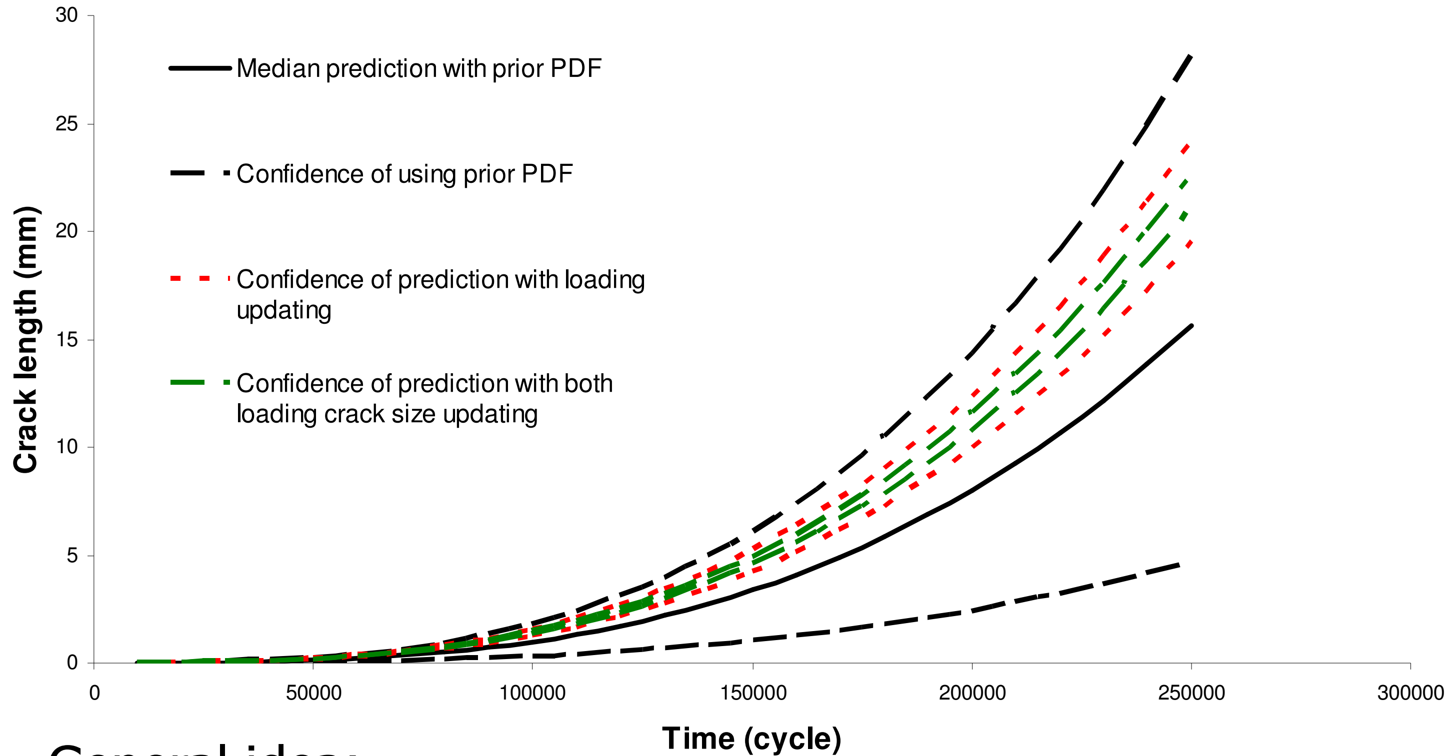
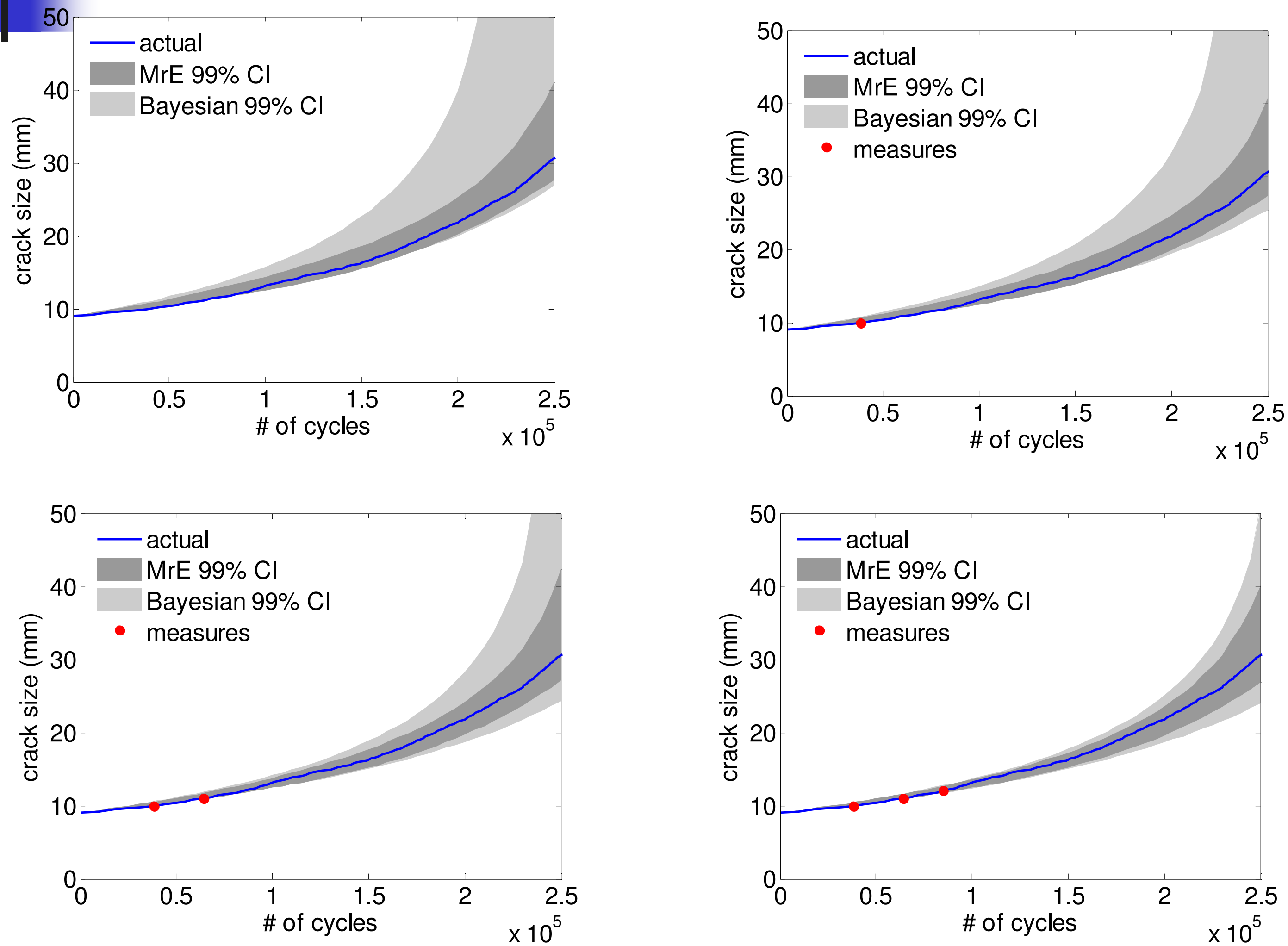
Uncertainty reduction via Bayesian updating



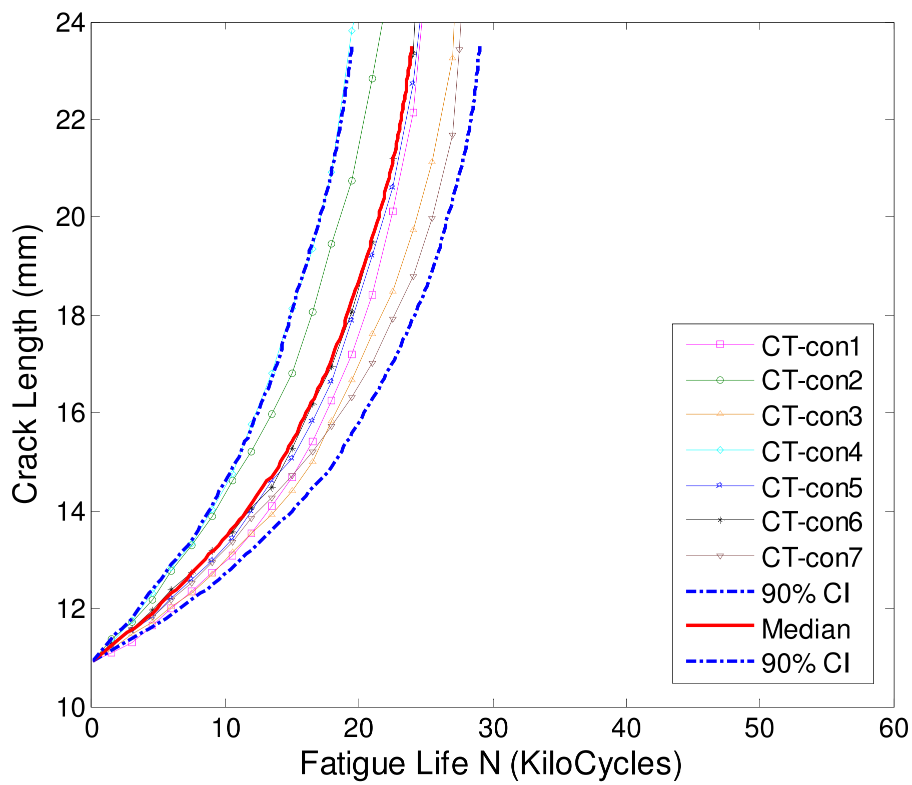
General idea:

Sequential Bayesian updating to reduce loading uncertainty and material uncertainty

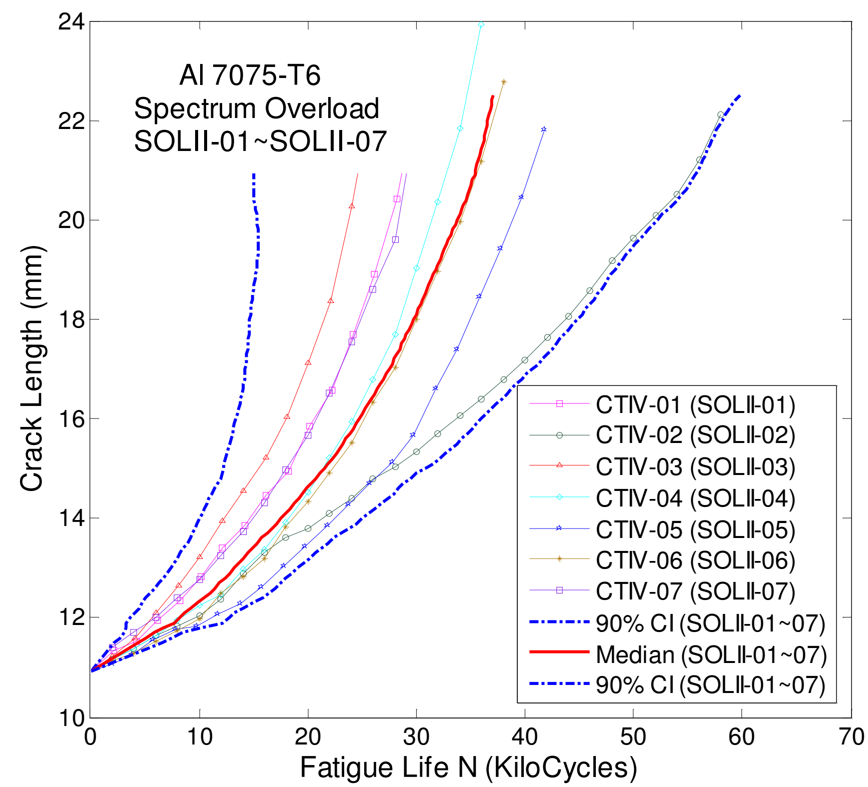
Case study 1 – constant amplitude loading



Case study 2 – Al 7075-T6 under random spectrum loadings

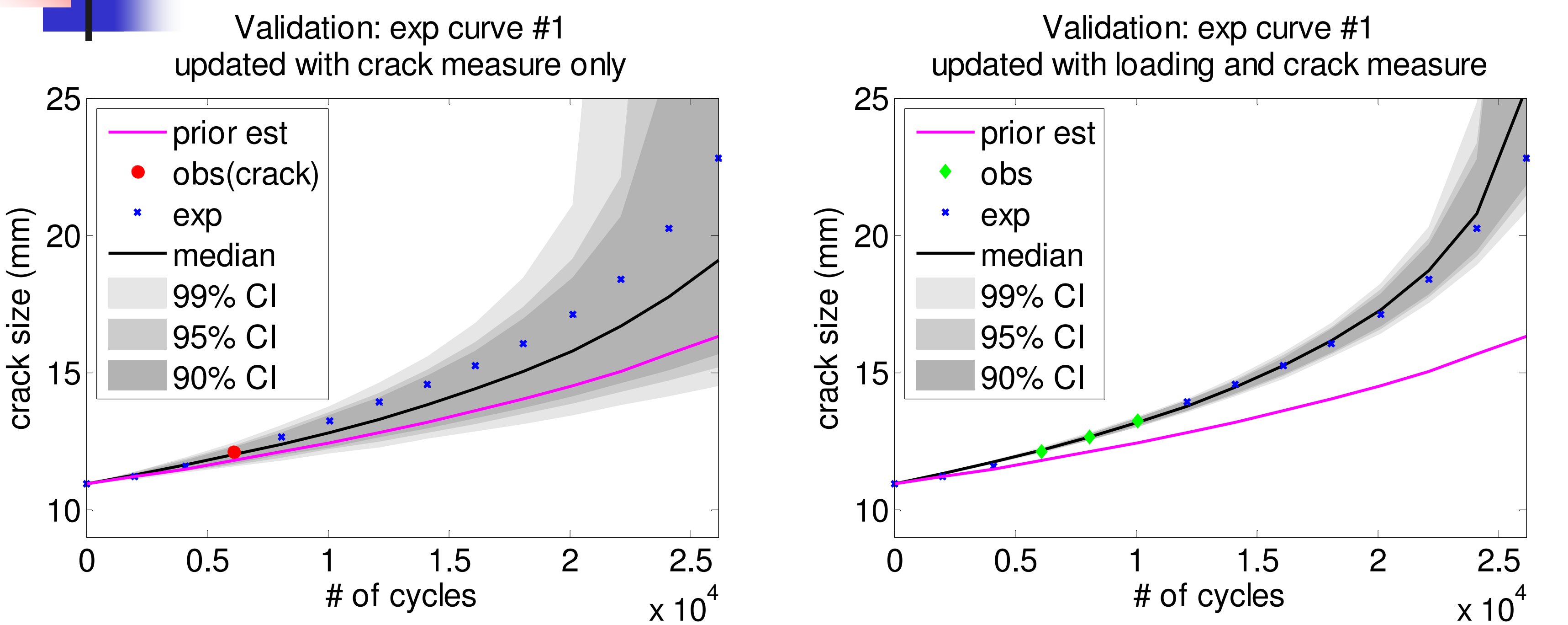


Constant amplitude loading data for calibration



Validation data under random variable amplitude loading

Effect of sequential updating using both loading and crack size measure



- Significant reduction of median bias and confidence bounds is observed
- Information fusion is very effective and both uncertain loading and material properties are included
- Modeling error and measurement noise are also included

Summary

- Bayesian/MRE updating provides a rigorous way to fuse "existing knowledge" (prior) and "new information" (observations)
- Both damage measure and loading measure can be included in the fusion process
- Very effective in uncertainty reduction of prognostics
- Effect of prior distribution may be significant when the number of observations is not sufficient large

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Verification and validation

- Visual graphical comparison is useful but does not provide quantitative judgment of the investigated prognostic algorithms
- V & V provides a rigorous way to evaluate the model performance and uncertainty management methodology

  - Metrics to evaluate the "absolute" prognostics algorithms' performance (Prognostics Metrics)

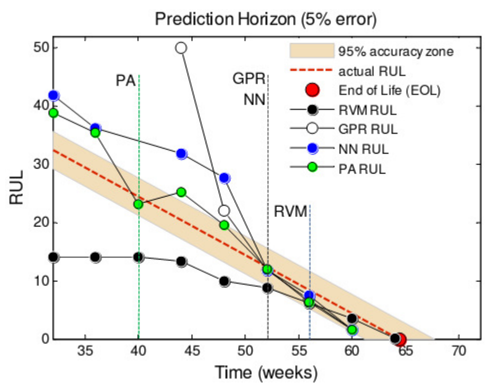
  - Metrics to evaluate the "relative" prognostics algorithms' performance (Bayes Factor)

Prognostics metrics - 1

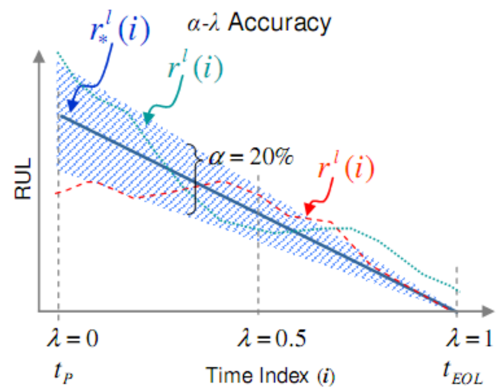
- Classical metrics
  - Based on statistical analysis, a large number of samples are required
  - Difficult to describe the prognosis performance over time
- Prognostics-based metrics [1]
  - Designed to describe how well an algorithm improve over time
  - Not based on statistics, no sample required
  - 4 metrics: Prognostic Horizon (PH), α - λ accuracy, Relative Accuracy (RA), Convergence

[1] A. Saxena, J. Celaya, B. Saha, S. Saha, and K. Goebel (2009). Evaluating algorithm performance metrics tailored for prognostics. IEEE Aerospace conference, 7-14 March 2009, pp. 1-13.

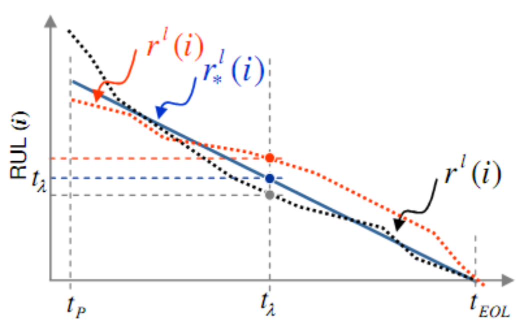
Prognostics metrics - 2



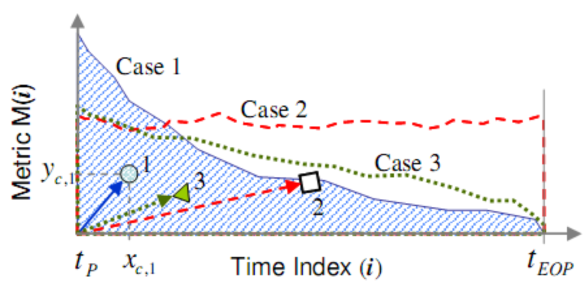
Prognostic Horizon (PH)



α - λ Accuracy (α - λ)



Relative Accuracy (RA)



Convergence

PH:
Length of time before end-of-life (EOL) when predictions are within specified $\pm\ \alpha\ \%$ accuracy limits

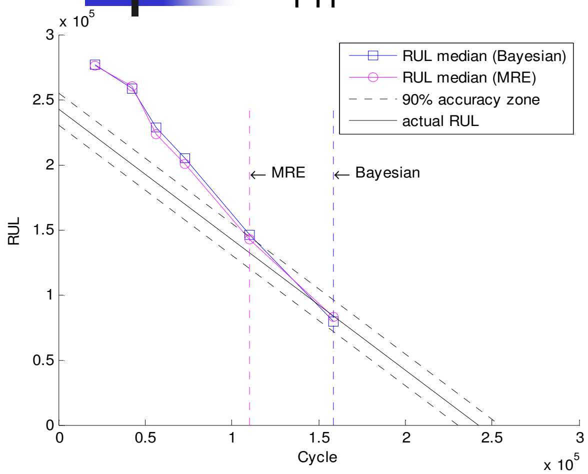
α - λ :
Determines whether predictions are within ± α % accuracy cone at a time instant specified by λ

RA:
Quantify accuracy w.r.t. the actual remaining useful life (RUL) at a given time instant specified by λ

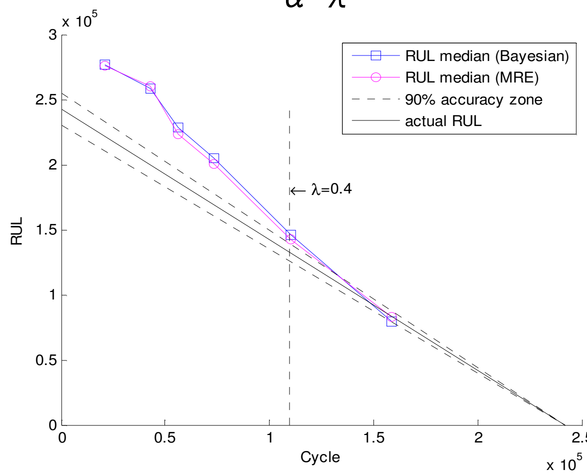
Convergence:
Quantify the rate at which any metric of interest improves to reach its desired value as time passes by

Case study: RUL prediction of fatigue crack growth

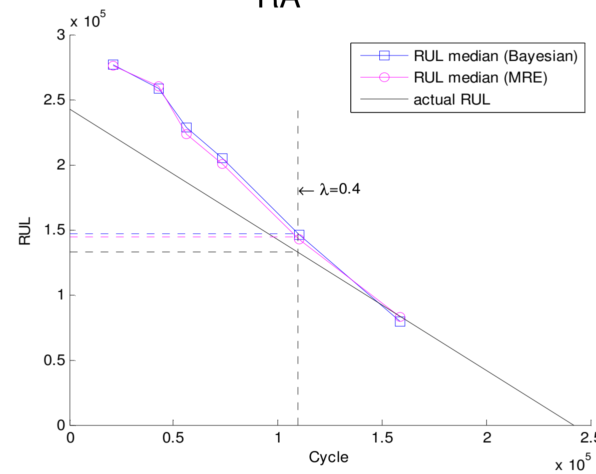
PH



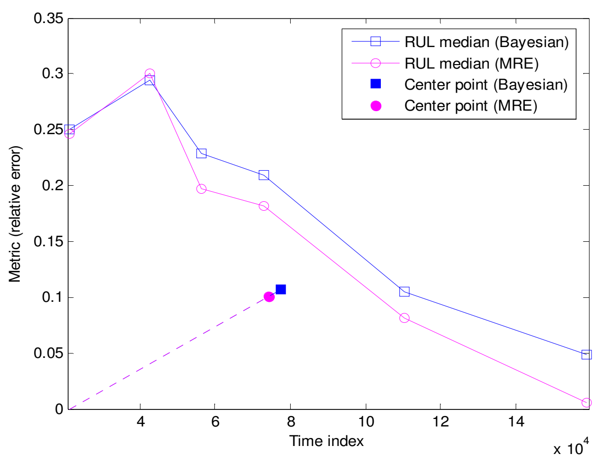
α - λ



RA



Convergence



Metric	MRE	Bayesian
MAPE	8.66	10.93
Average Bias (cycles)	10956.27	14051.92
STD(cycles)	7628.77	9115.78
MSE(cycle^2)	178.23 x 10^6	280.5 x 10^6
PH $_{\alpha=10\%}$	183283	169451
RA $_{\lambda=0.4}$	0.92	0.89
CRA $_{\lambda=0.4}$	0.89	0.87
Convergence (RA)	74365.72	77349.24

Model performance comparison

- Competing models exist for many engineering problems

  - mechanism is not clear

  - each model represents a part of underlying physics/population behavior

  - hybrid data-driven and model-based approaches
- Automatic model selection and model fusion based on new observations
- Include uncertainties in the model performance comparison
- Protect against overfitting

Bayes factor-based model selection and fusion

- Bayes factor is the likelihood ratio of different models

$$B_{i,j} = p(X \mid M_i) / p(X \mid M_j)$$

- A value of $B > 1$ indicates that M_i is more strongly supported by the data under consideration than M_j
- A decisive model selection can be made if B is very large or model averaging/fusion if not

Problem formulation

- A finite set of model candidates

$$\left\{\cup_{i=1}^{n} \boldsymbol{\Theta}_i^{(k_i)}, \boldsymbol{M}\right\} = \left\{(\boldsymbol{\theta}_1^{(k_1)}, M_1^{(k_1)}), (\boldsymbol{\theta}_2^{(k_2)}, M_2^{(k_2)}), \ldots, (\boldsymbol{\theta}_n^{(k_n)}, M_n^{(k_n)})\right\}$$

- Treat models as random variables with associated probabilities

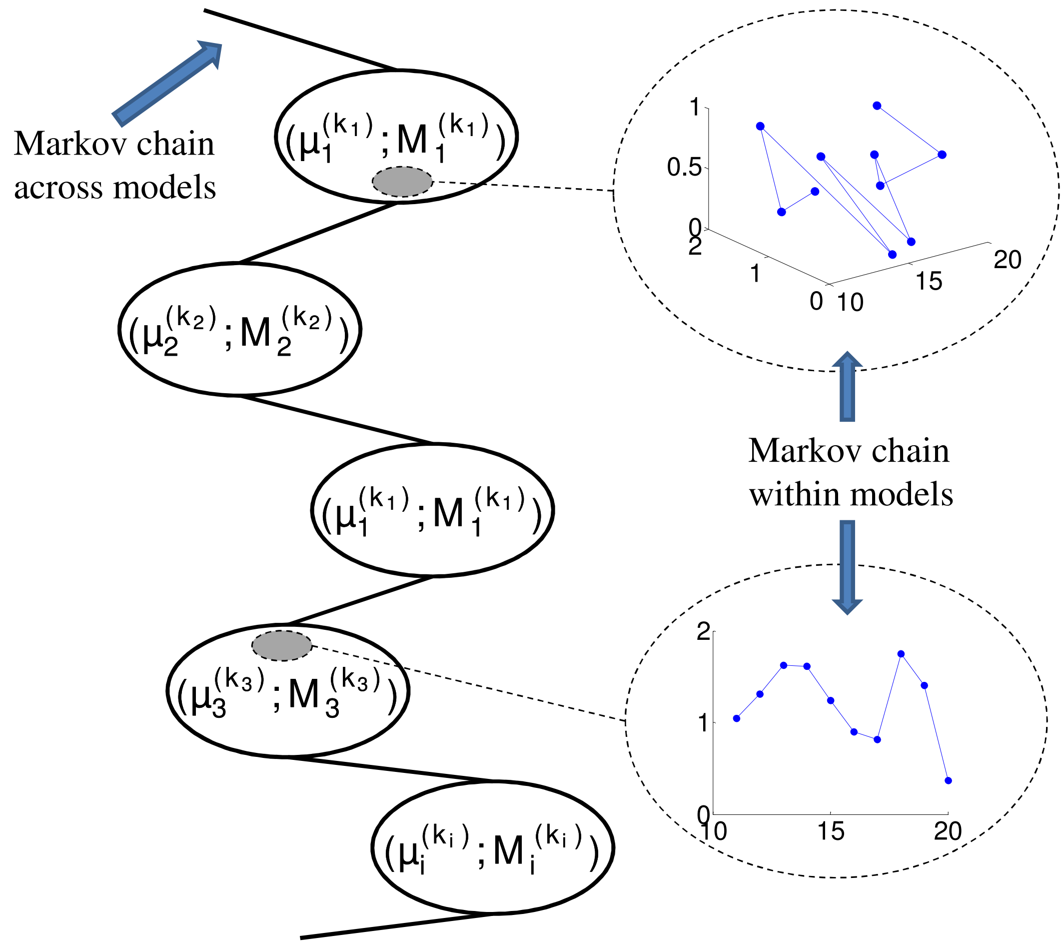
$$P(M_i \mid X) = \frac{P(M_i)P(X \mid M_i)}{P(X)}$$

- An event X, total probability

$$P(X) = \sum_{i=1}^{n} P(M_i^{(k_i)})P(X|M_i^{(k_i)}) = \sum_{i=1}^{n} P(M_i^{(k_i)}) \int_{\boldsymbol{\Theta}_i^{(k_i)}} p(\boldsymbol{\theta}_i^{(k_i)}|M_i^{(k_i)})p(X|\boldsymbol{\theta}_i^{(k_i)}, M_i^{(k_i)})\mathrm{d}\boldsymbol{\theta}_i^{(k_i)}$$

Computational algorithm

- Trans-dimensional MCMC
  - Different models are associated with different model structures (e.g., number of parameters)
  - Multiple runs are required using the classical MCMC
  - Trans-dimensional MCMC finds a mapping transformation and only need one MCMC simulation



Case study: competing models for fatigue crack growth analysis

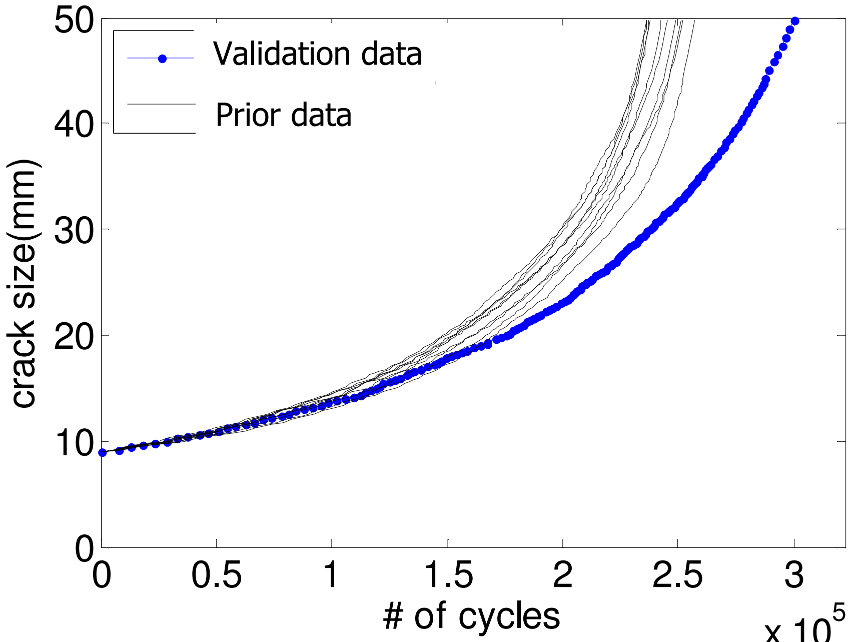
- Paris' model $$\frac{da}{dN} = C(\Delta K)^m$$
- Forman's model

$$\frac{da}{dN} = C(\Delta K)^m /((1-R)K_c - \Delta K)$$

- McEvily's model

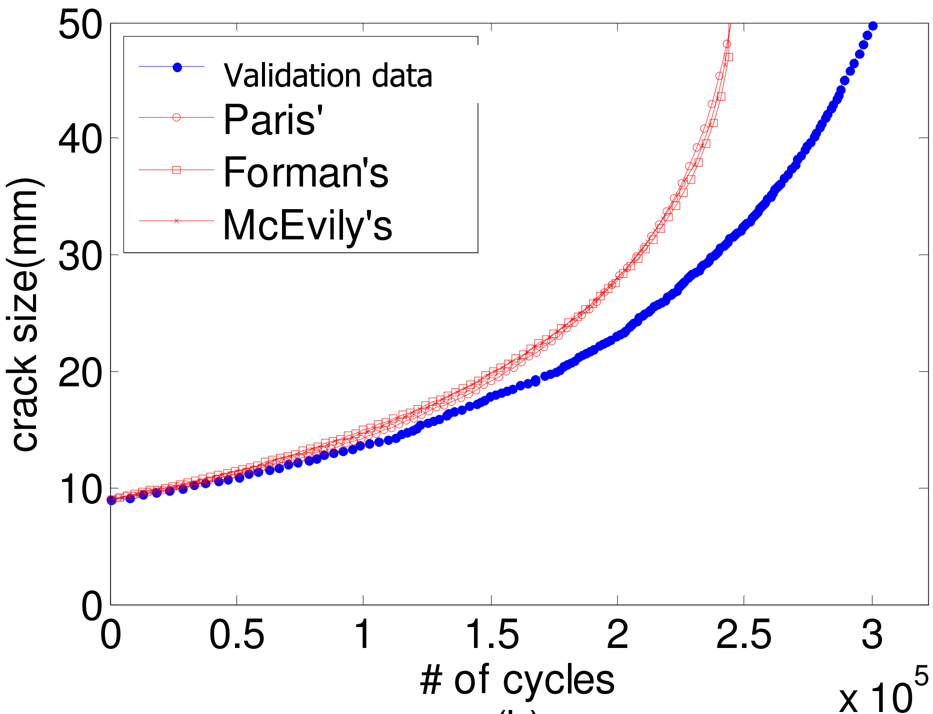
$$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^2 (1 + \frac{\Delta K}{K_c - K_{\max}})$$

Case study: Al 2024-T3 data and prior distribution



(a)

Raw data



(b)

Point estimate
using prior fitting

Results

Bayes factors

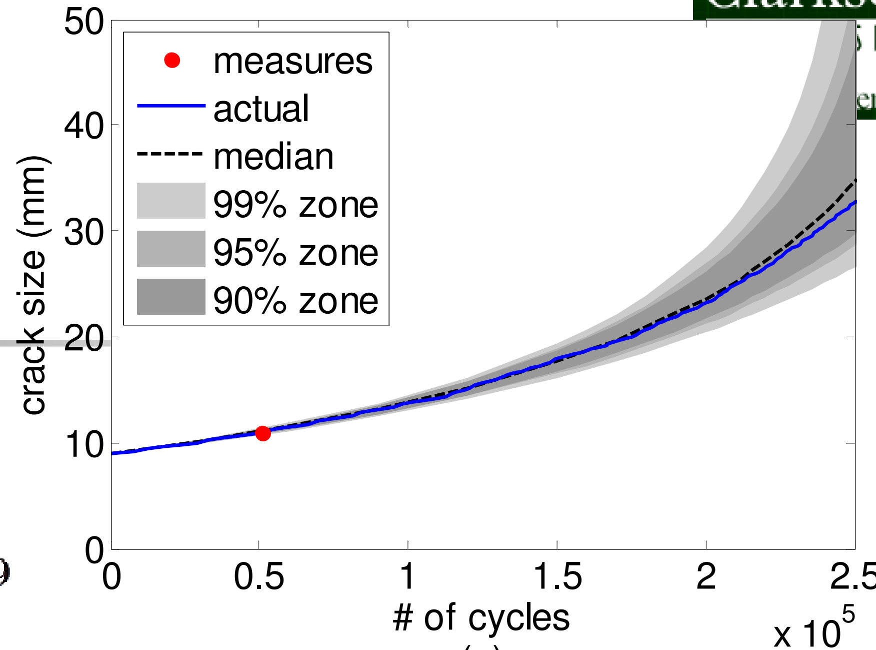
$$1^{st}\ \text{update}: \begin{cases} B_{p,f} = \langle P(M_p)\rangle / \langle P(M_f)\rangle = 12.046 \\ B_{m,f} = \langle P(M_m)\rangle / \langle P(M_f)\rangle = 23.689 \\ B_{m,p} = \langle P(M_m)\rangle / \langle P(M_p)\rangle = 1.967 \end{cases}$$

$$2^{nd}\ \text{update}: \begin{cases} B_{p,f} = \langle P(M_p)\rangle / \langle P(M_f)\rangle = 40.413 \\ B_{m,f} = \langle P(M_m)\rangle / \langle P(M_f)\rangle = 80.702 \\ B_{m,p} = \langle P(M_m)\rangle / \langle P(M_p)\rangle = 1.997 \end{cases}$$

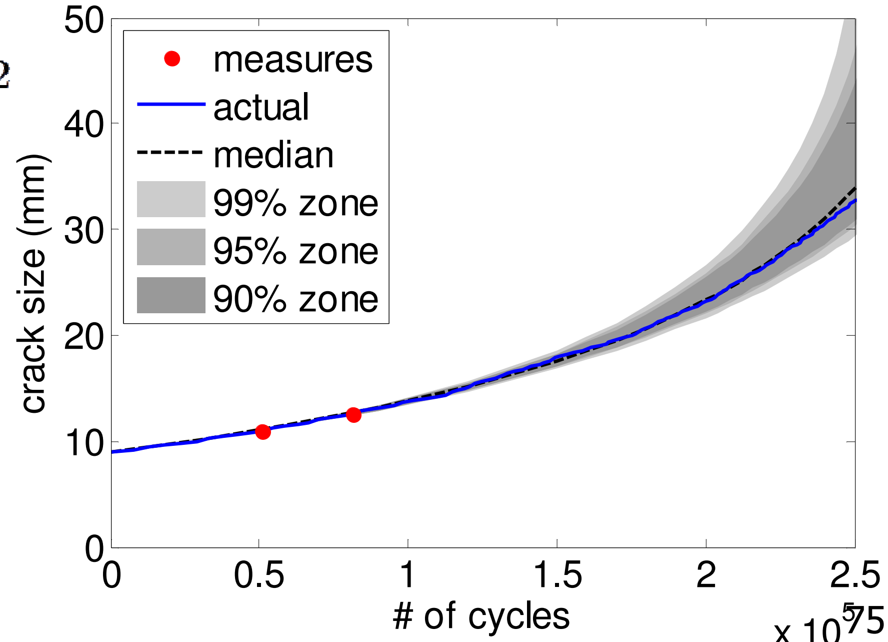
Current data strongly support McEvily's/Paris's model over Forman's model

Slight preference of McEvily's model over Paris' law

Above statement is only valid for the investigated data



(a)



(b)

Conclusions

- General uncertainty management framework is presented
- Small time scale mechanism-based crack growth model
- Modeling uncertainty and covariance structure in uncertainty quantification
- Three efficient algorithms for uncertainty propagation
- Generalized Bayesian methodology for uncertainty updating
- Metrics-based model selection and validation

Acknowledgement

- Collaborators

NASA Ames:
Kai Goebel, Abhinav Saxena, Kevin Wheeler, Jose Celaya, Bhaskar Saha

NASA Langley:
Thiagarajan (Krish) Krishnamurthy, Tzi-Kang (John) Chen, Rick Ross

Vanderbilt University:
Sankaran Mahadevan

Thanks!

Questions?