POLITECNICO DI MILANO

Particle filters for prognostics

Piero Baraldi[1], Francesco Di Maio[1], Enrico Zio[2,1]

[1]Dipartimento di Energia, Politecnico di Milano, Italy
[2]Chair on Systems Science and the Energetic challenge, European Foundation for New Energy-EDF, Ecole Centrale Paris and Supelec, France

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Outline

- Prognostics
- Model-based prognostics
- Particle filtering for degradation state estimate
- Particle filtering for RUL estimate
- Application
  - Maintenance planning

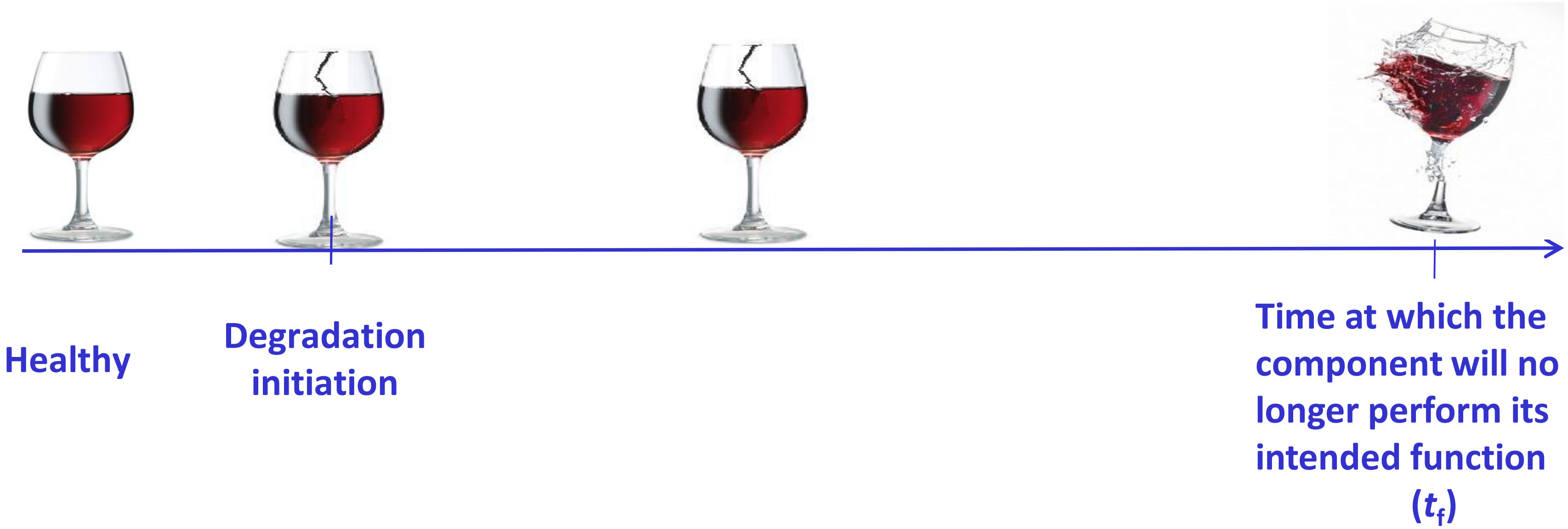
- Prognostics
  - What is it?
  - Sources of information
  - Prognostics in practice
  - Prognostic approaches

Prognostics: the degrading component

Prognostics: the degrading component

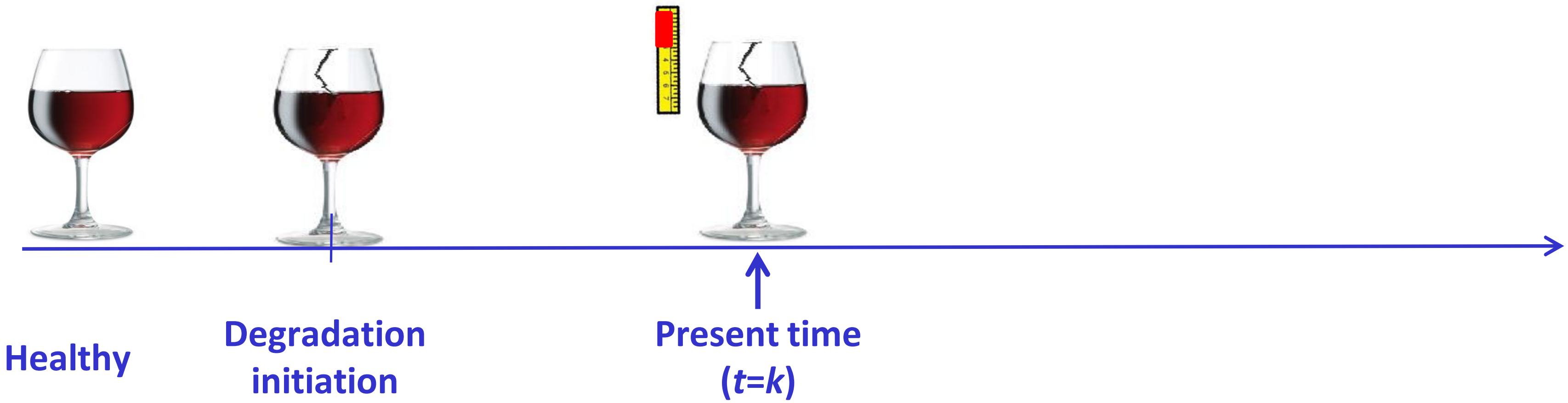
Prognostics: objectives

Evolution toé failure



Prognostics: objectives

Evolution toé failure

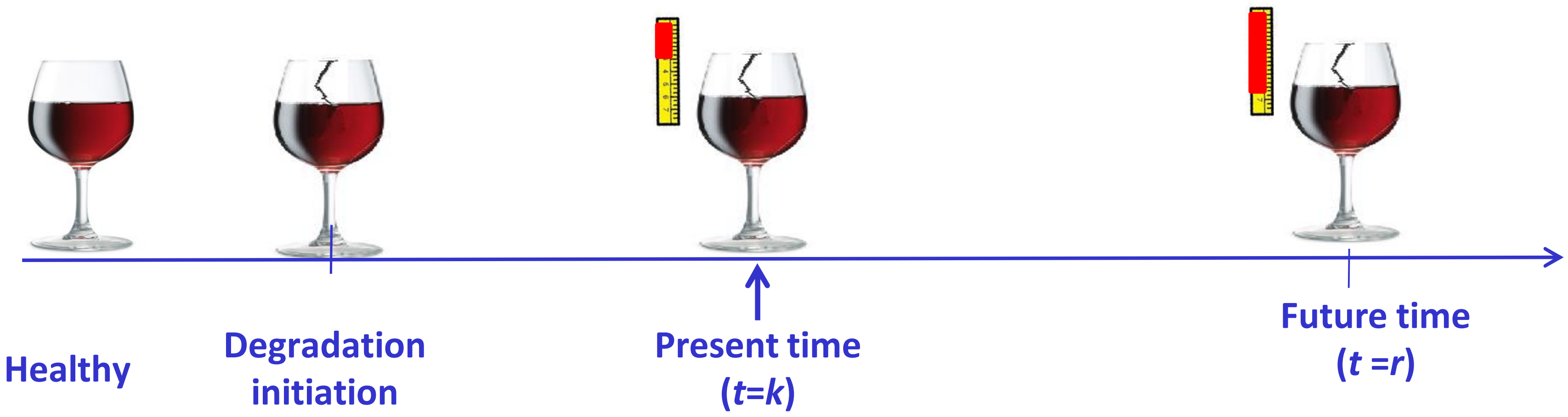


Our objectives:

1. **Estimate the component degradation at a the present time *t* =*k***

Prognostics: objectives

Evolution toé failure

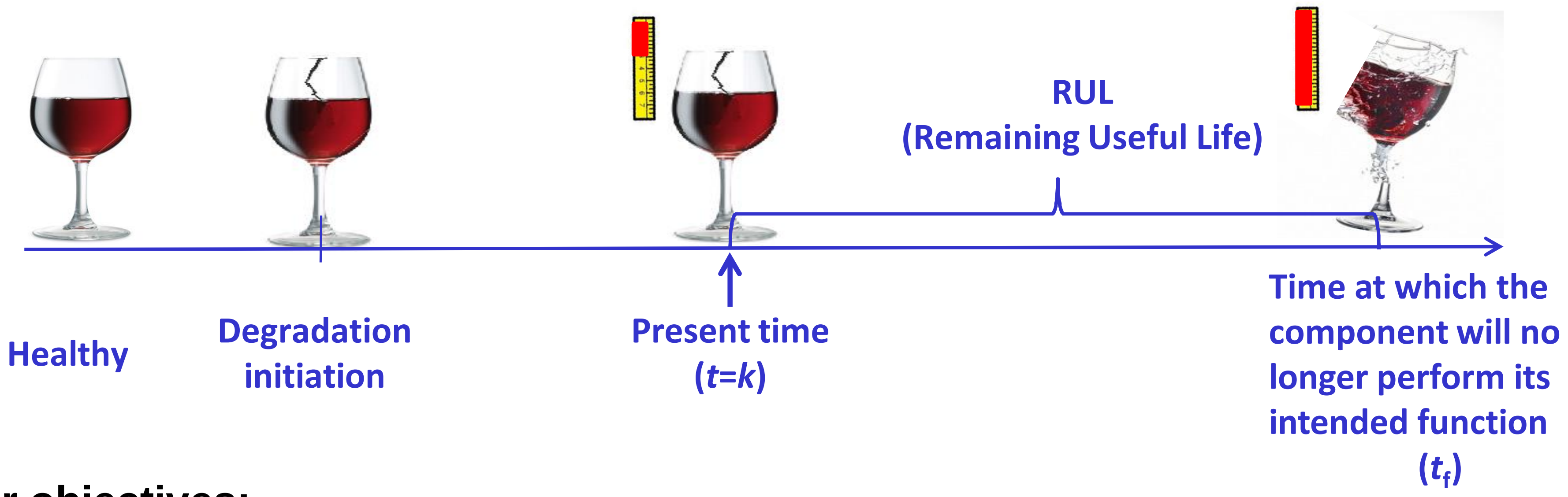


Our objectives:

1. **Estimate the component degradation at a the present time *t* =*k***
2. **Estimate the component degradation at a future time *r* > *k***

Prognostics: objectives

Evolution toé failure



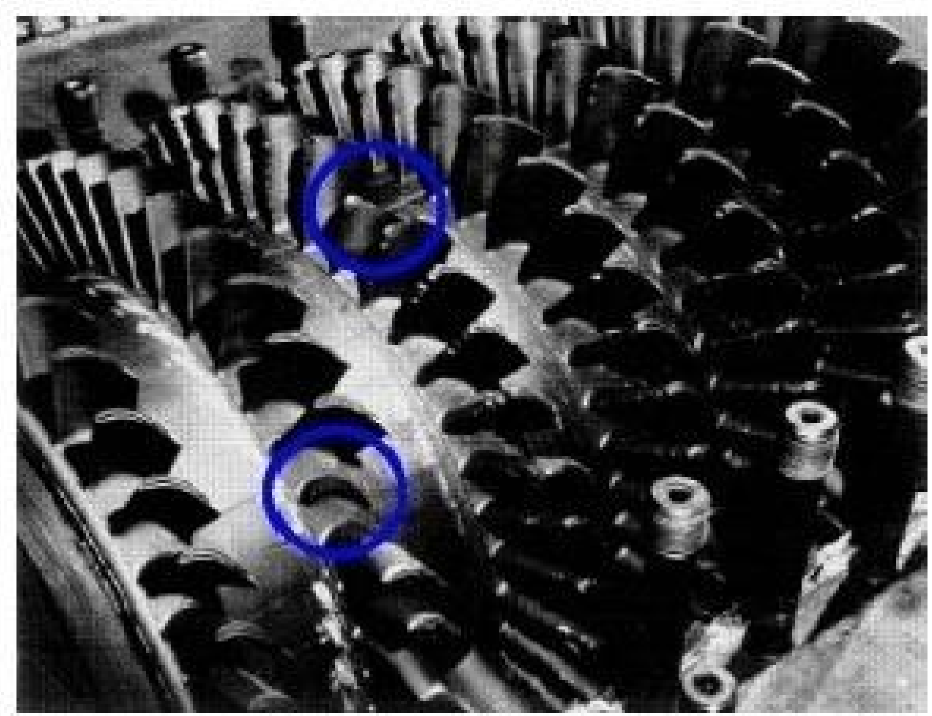
Our objectives:

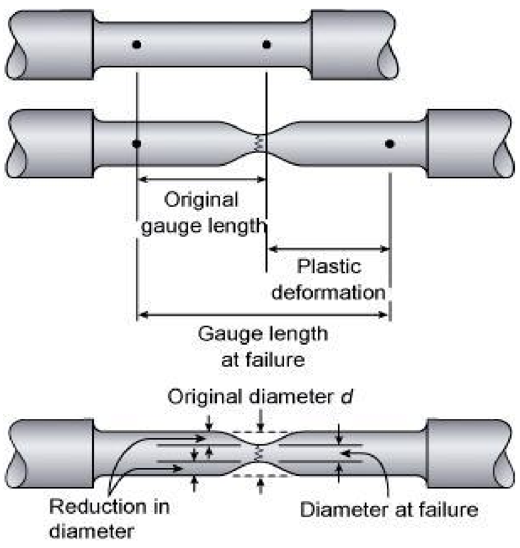
1. **Estimate the component degradation at a the present time $t = k$**
2. **Estimate the component degradation at a future time $r > k$**
3. **Estimate the component Remaining Useful Life (RUL) = $t_f - k$**

- Prognostics
  - What is it?
  - Prognostics in practice
  - Sources of information
  - Prognostic approaches

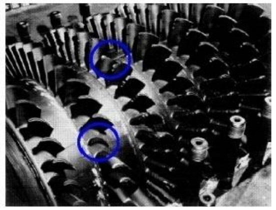
Turbine blade

Component: turbine blade
Degradation mechanism: creeping



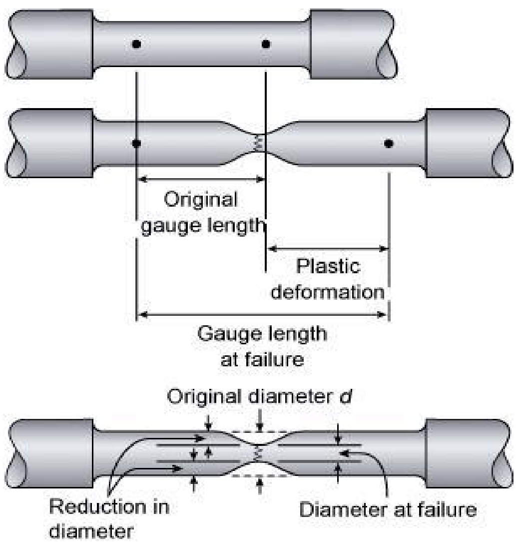


Definition of a degradation indicator



Component: turbine blade
Degradation mechanism: creeping

Degradation indicator: blade elongation $x(t) = \frac{\text{Length}(t) - \text{initial length}}{\text{initial length}}$



Our objectives:

1. Estimate the blade degradation at the present time $t = k$
2. Estimate the blade degradation at a future time $r > k$
3. Estimate the component Remaining Useful Life (RUL)

- Prognostics
  - What is it?
  - Prognostics in practice
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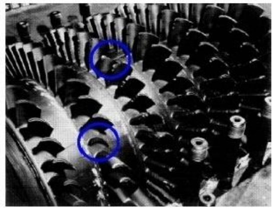
Sources of information for prognostics

Å A physical model of the degradation process (dynamic law describing the evolution of the degradation indicator, x):

Norton law for creep growth

$$\frac{dx}{dt} = A\exp\left(-\frac{Q}{RT}\right)\varphi^{n}$$

Arrhenius law



x = blade elongation

T = temperature

$ű = K d_r^{\,2}$ = applied stress

d_r = rotational speed

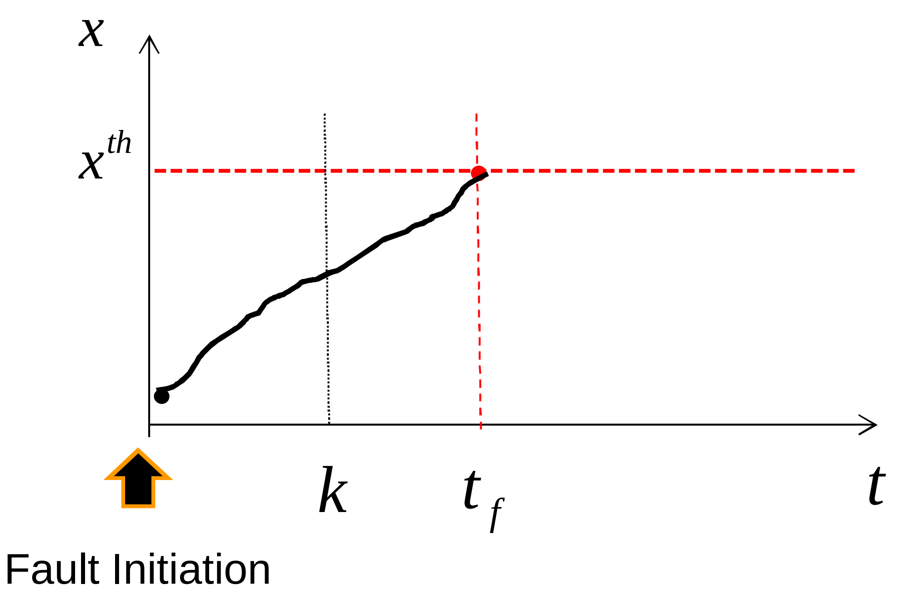
External/operational conditions

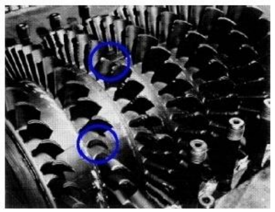
A, Q, n = equipment inherent parameters

Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure: x^{th}

«A blade is discarded when the elongation, *x*, reaches 1.5%»

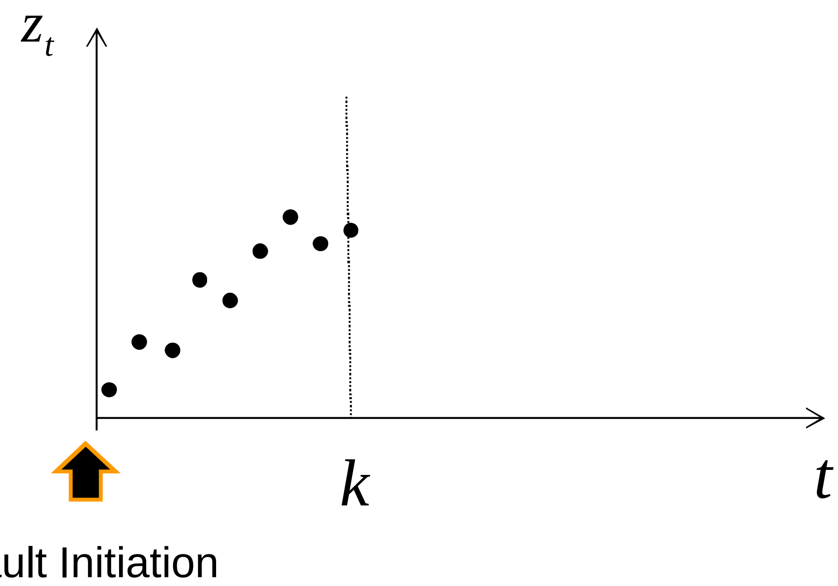


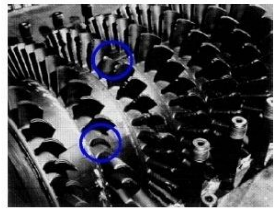


Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure
- A sequence of observations, related to the component degradation, collected from the degradation initiation to the present time

Elongation measurements = past evolution of the degradation indicator



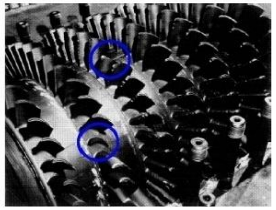


Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure
- A sequence of observations, related to the component degradation, collected from the degradation initiation to the present time
- Measurement equation: $z = h(x, \nu)$

Random noise with known distribution

$$z = x + \nu$$
$$\nu \propto N\left(0, \sigma^2\right)$$



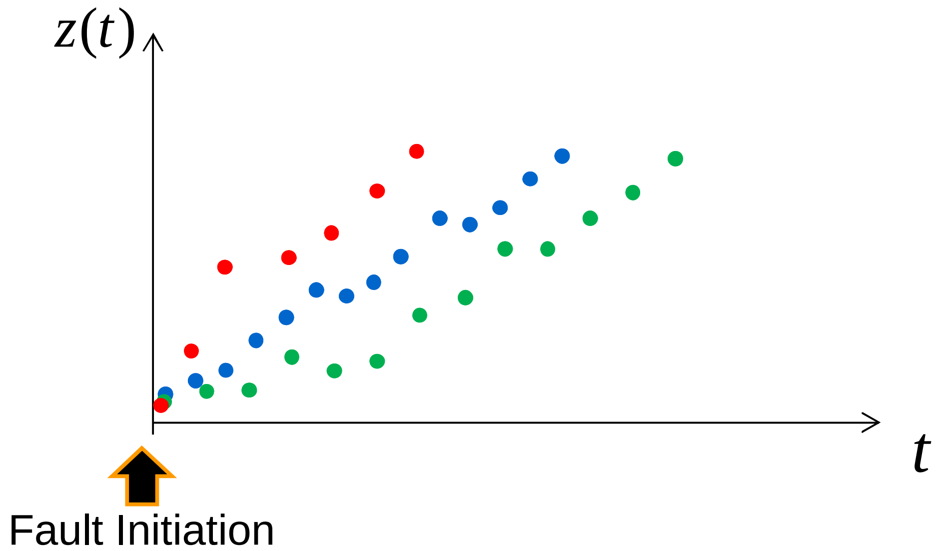
Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure
- A sequence of observations, related to the component degradation, collected from the degradation initiation to the present time
- Measurement equation
- Life durations of a set of similar components which have already failed:

$$t_1^f, t_2^f, \ldots, t_s^f$$

Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure
- A sequence of observations, related to the component degradation, collected from the degradation initiation to the present time
- Measurement equation
- Life durations of a set of similar components which have already failed
- A set of observations performed on a set of similar components from degradation initiation to failure



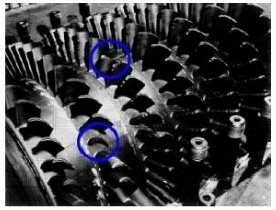
Sources of information for prognostics

- A physical model of the degradation process
- Threshold of failure
- A sequence of observations, related to the component degradation, collected from the degradation initiation to the present time
- Measurement equation
- Life durations of a set of similar components which have already failed
- A set of observations performed on a set of similar components from degradation initiation to failure
- External/operational conditions $u_1, u_2, \ldots, u_k, u_{k+1}, \ldots$

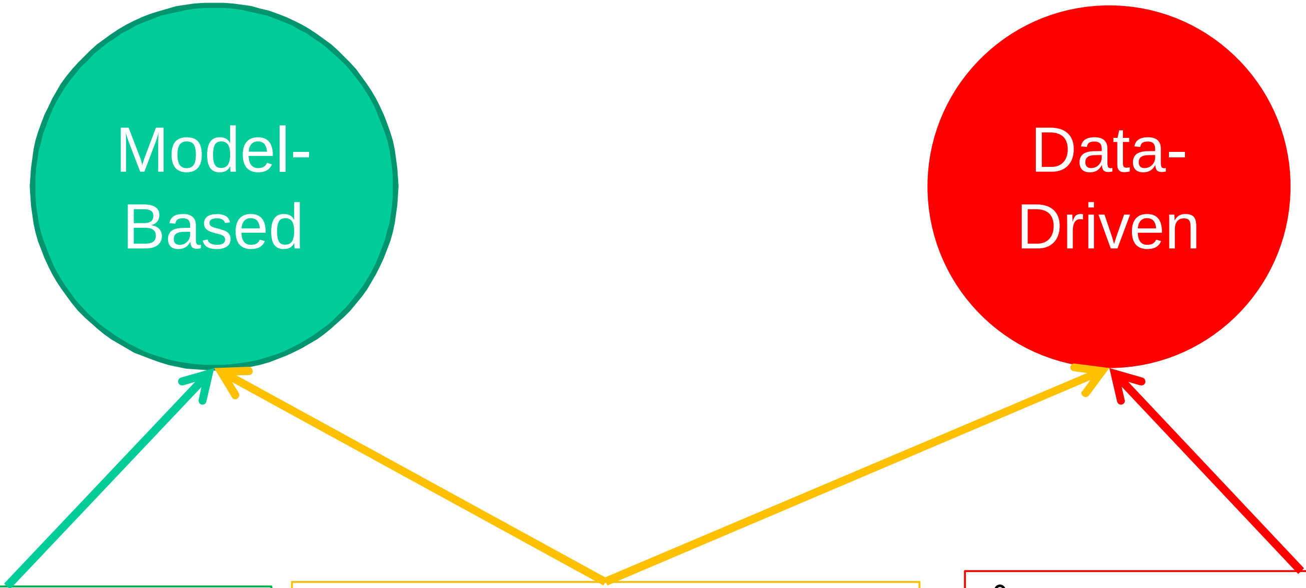
Past, present and future time evolution of:

T **= temperature**

$\boldsymbol{d_r} \equiv$ **rotational speed**



- Prognostics
  - What is it?
  - Prognostics in practice
  - Sources of information
  - Prognostic approaches



- A physical model of the degradation process
- Measurement equation

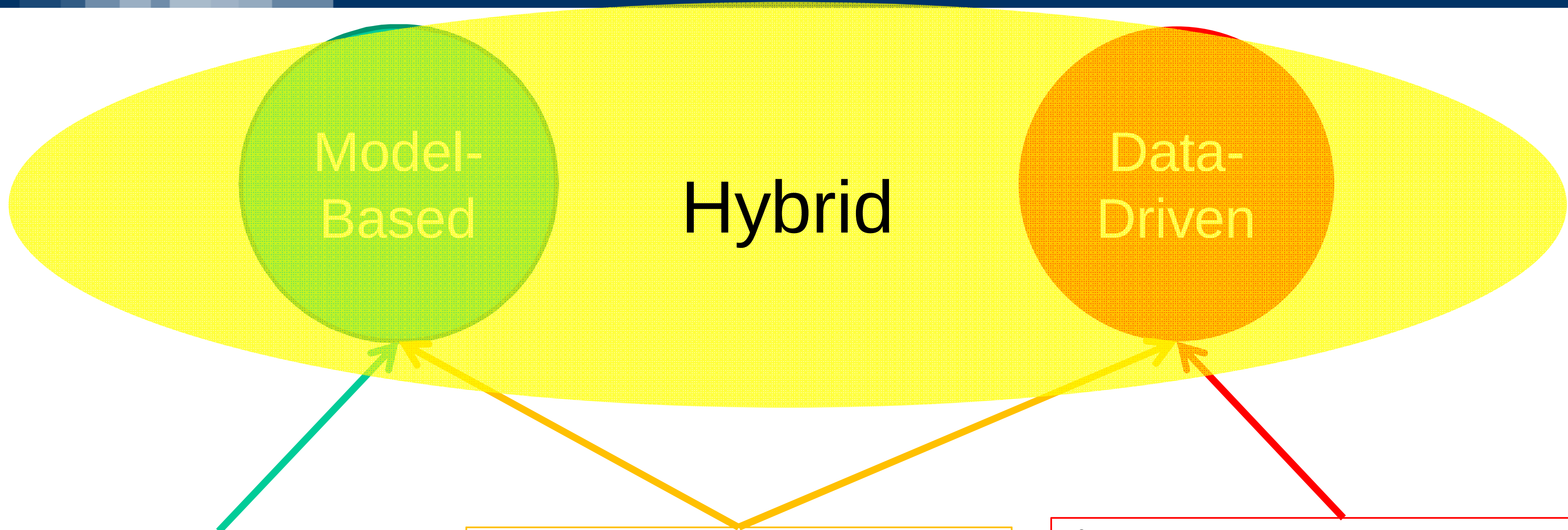
- A sequence of observations related to the component degradation collected from the degradation initiation to the present time
- A threshold of failure
- External/operational conditions

Degrading component

- Life durations of a set of similar components which have already failed
- A set of observations performed on a set of similar components from degradation initiation to failure

Similar components

Prognostic approaches



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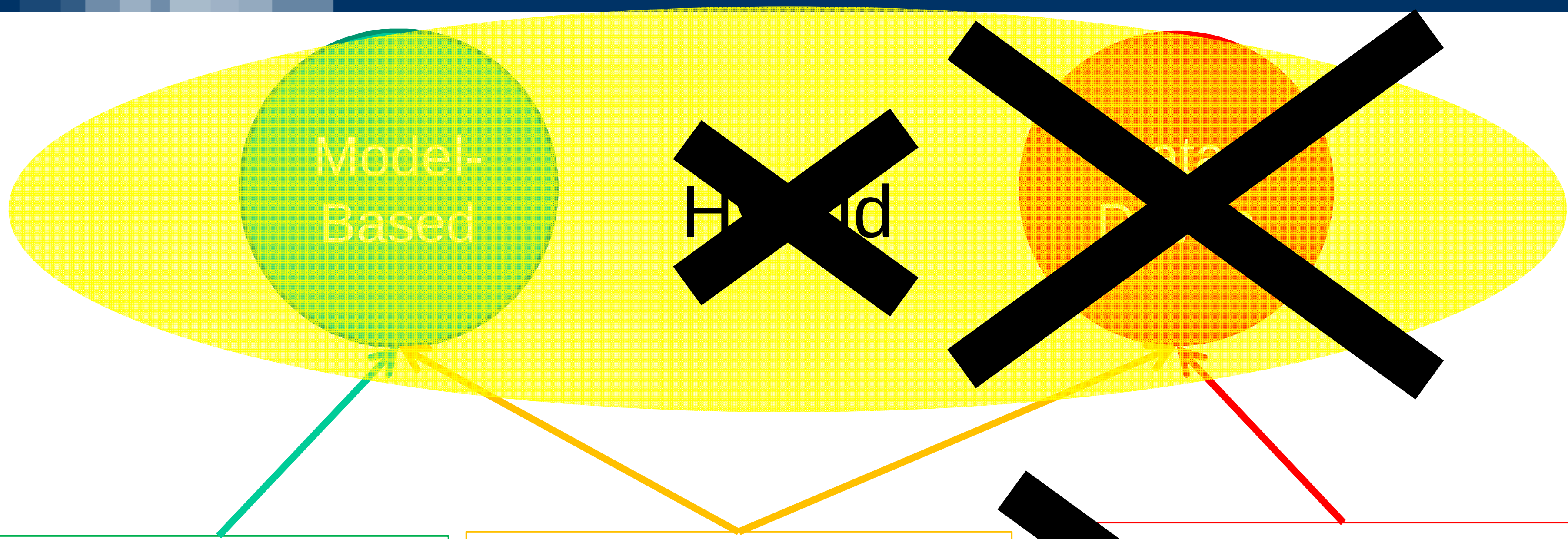
Degrading component

Similar components

Prognostic approaches



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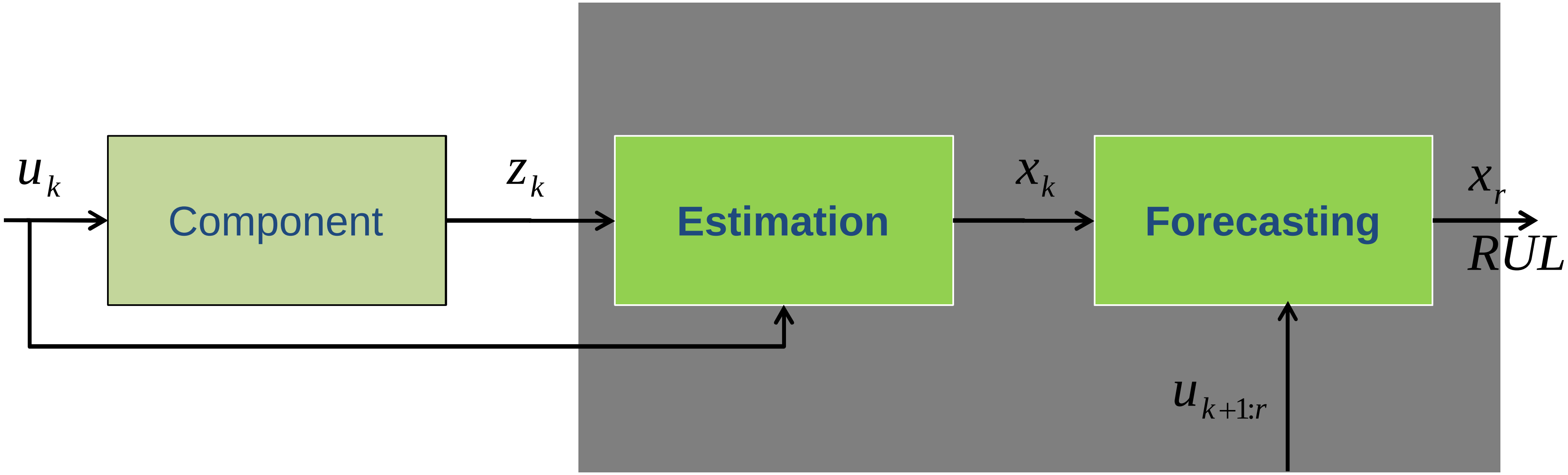
Degrading component

- [illegible]ife durations of a set of si[illegible]ar components wh[illegible] have [illegible]dy failed
- A set of o[illegible]ns performed [illegible]t of similar compo[illegible]s from de[illegible]ation initiation [illegible]ure

Similar components

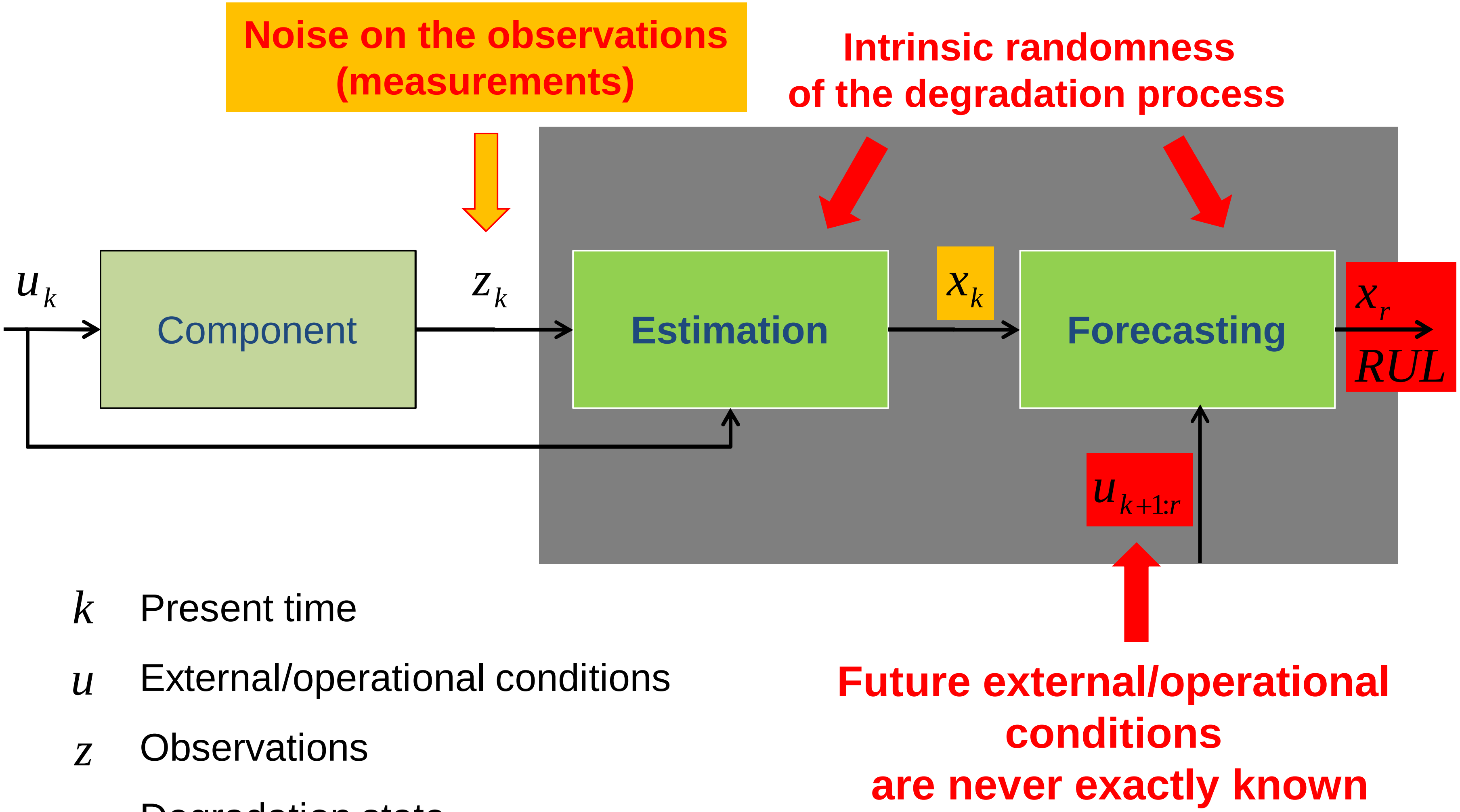
- Model-based prognostics
  - The filtering problem
  - The forecasting problem

Model-based prognostics: the methodology



k	Present time
u	External/operational conditions
z	Observations
x	Degradation state

Main sources of uncertainty



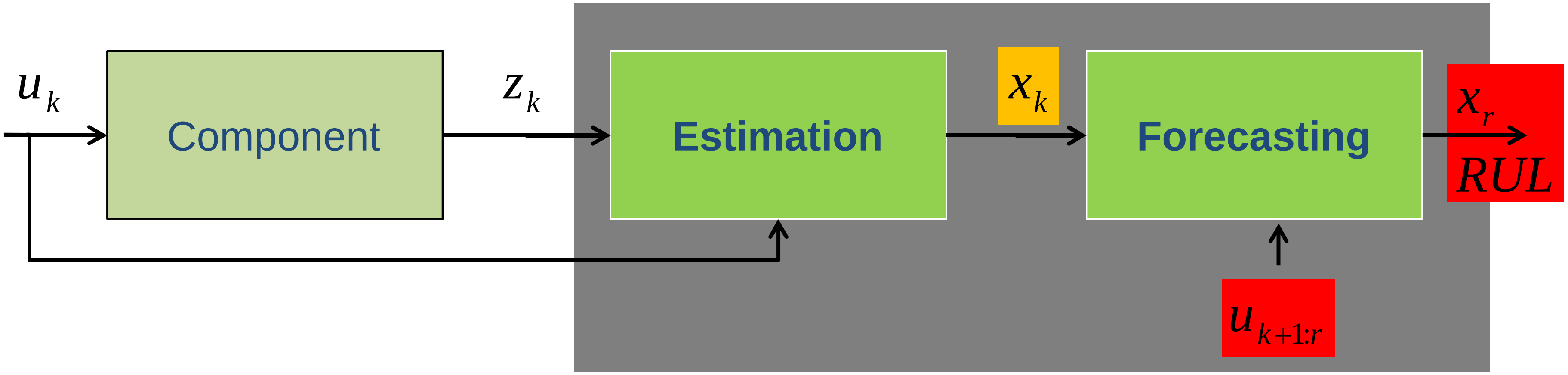
k Present time

u External/operational conditions

z Observations

x Degradation state

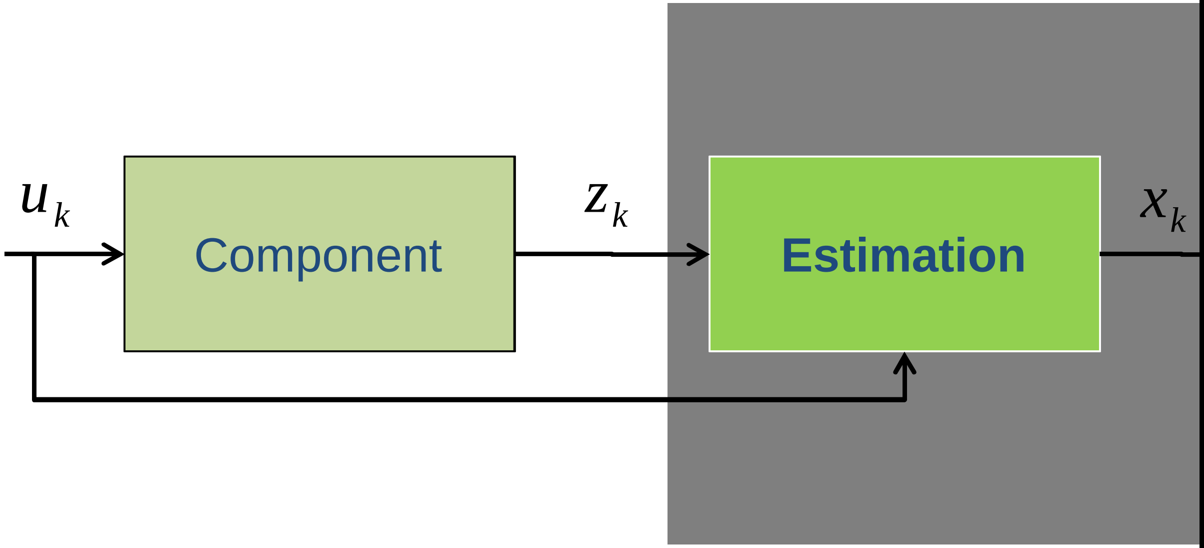
Prognostics = Filtering + Forecasting



1. The filtering problem: to estimate the degradation state, x_k , at the present time
2. The forecasting problem:
   - to predict the degradation state, x_r , at a future time *r*
   - to predict the component RUL

- Model-based prognostics:
  - The filtering problem
  - The forecasting problem

The Filtering Problem



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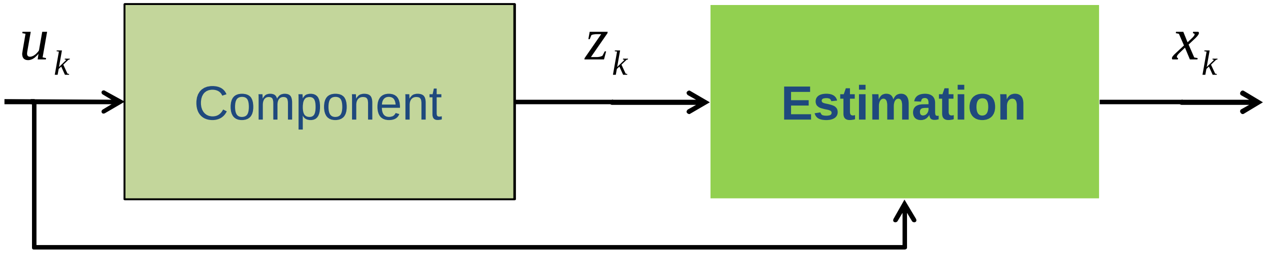
k Present time

u External/operational conditions

z Observations

x Degradation state

Problem Setting



- Physical model of the degradation process

$$x_k = f_k\left(x_{k-1}, \omega_{k-1}\right)$$

Time-discrete, hidden Markov process

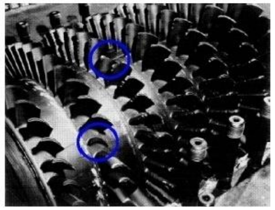
  - x = **hidden** degradation state
  - ω = random **process noise**
  - *f* = physical model of the degradation process (non-linear dynamic law)
  - *k* = time step index

- Measurement equation:

$$z_k = h\left(x_k, v_k\right)$$

  - υ = random **measurement noise**
  - *h* = non-linear measurement equation

The filtering problem in practice
(Physical model of the degradation process)



- *x* = hidden degradation state (blade elongation)
- T_0, ϑ_0 = operational conditions
- $\omega_1, \omega_2, \omega_3$ = random process noises $\omega_i \propto N(0, \sigma_i^2)$
- *A*, *K* and *n* = constants related to the material properties

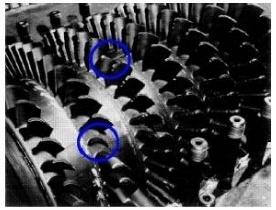
$$\frac{dx}{dt} = A \cdot \exp\left(- \frac{Q}{R \cdot (T_0 + \omega_1)} \right) \cdot \left(K \cdot (\vartheta_0 + \omega_2)^2 + \omega_3 \right)^n$$

Norton law for creep growth

Discretization of the dynamics

$$x_k = x_{k-1} + A \cdot \exp\left(- \frac{Q}{R \cdot (T_0 + \omega_1)} \right) \cdot \left(K \cdot (\theta_0 + \omega_2)^2 + \omega_3 \right)^n$$

The filtering problem in practice (Measurement Equation)

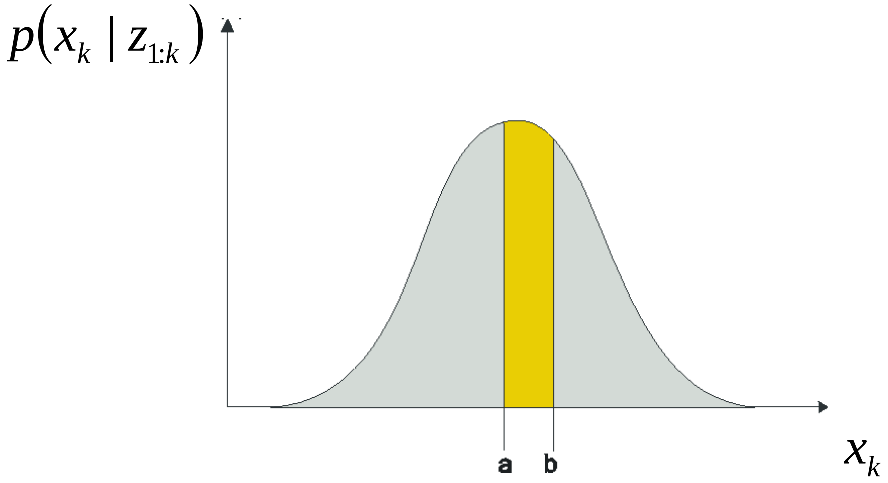


$$z_k = h(x_k, \nu_k) = x_k + \nu_k$$

- z_k = degradation observation (measure of the creep elongation)
- υ_k = igaussian measurement noise

The Bayesian framework

OBJECTIVE: $p(x_k \mid z_{1:k})$

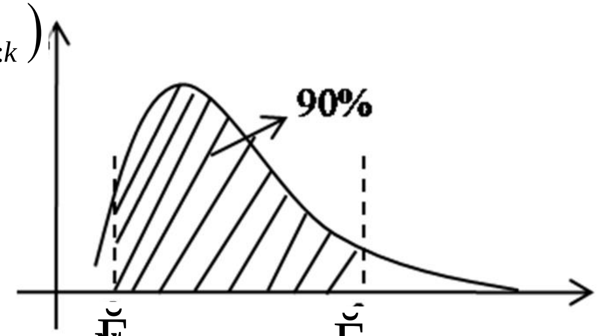


Å Interpretation of the bayesian probability $p(x_k \mid z_{1:k})$?

- conditional on the background knowledge: the noisy measurements $z_{1:k} = z_1, z_2, \ldots, z_k$
- subjective probability = **degree of belief** with regard to the hidden degradation state x_k

$$p\left(x_k \mid z_{1:k}\right)$$

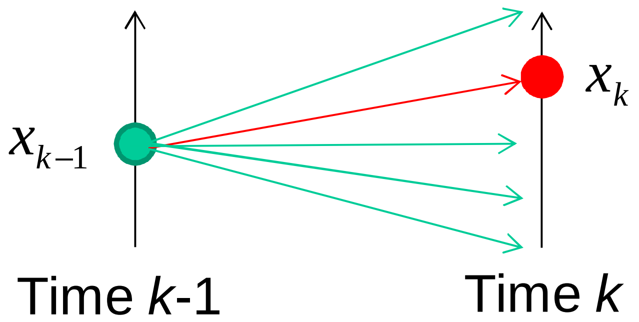
- state **mean** (estimate) $\hat{x}_k = \int p\left(x_k \mid z_{1:k}\right) \cdot x_k \, dx_k$
- state **variance** (uncertainty) $\hat{\sigma}^2_{x_k} = \int \left(x_k - \hat{x}_k\right)^2 \cdot p\left(x_k \mid z_{1:k}\right) dx_k$
- state **percentiles** $\hat{x}_5, \hat{x}_{95}$



The sequential solution (I)

- Let us assume that we know $p(x_{k-1} \mid z_{1:k-1})$ at time k-1

- Prediction stage: Chapman-Kolmogorov equation



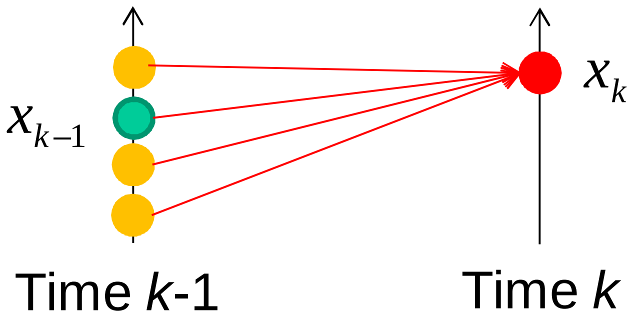
$$\ldots p(x_k \mid x_{k-1})\, p(x_{k-1} \mid z_{1:k-1}) \ldots$$

The sequential solution (I)

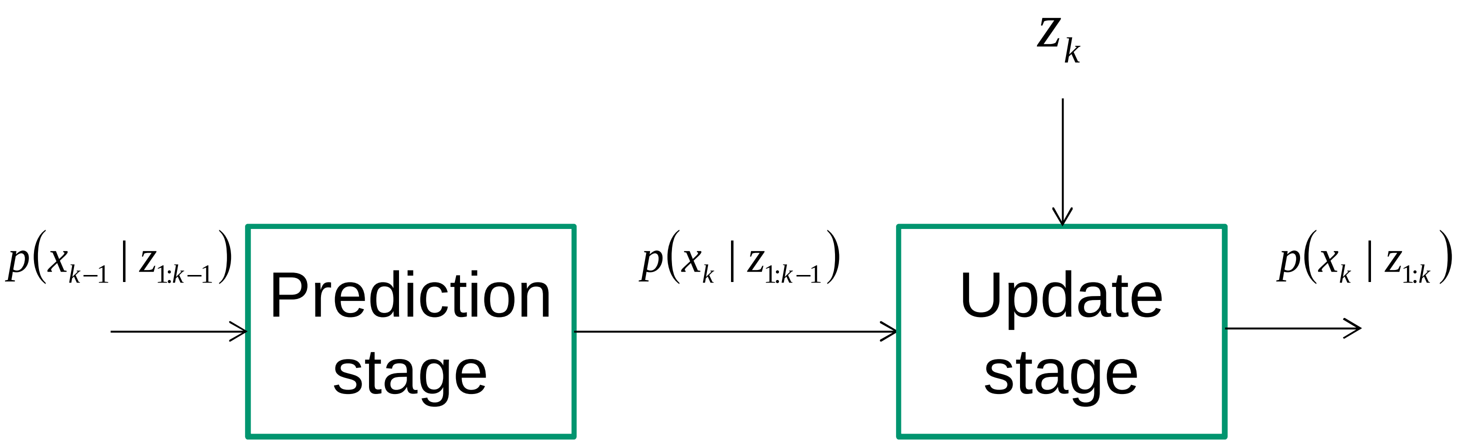
Å Let us assume that we know $p\left(x_{k-1} \mid z_{1:k-1}\right)$ at time *k*-1

Å Prediction stage: Chapman-Kolmogorov equation

$$p\left(x_k \mid z_{1:k-1}\right) = \int p\left(x_k \mid x_{k-1}\right) p\left(x_{k-1} \mid z_{1:k-1}\right) dx_{k-1}$$



The sequential solution (II)



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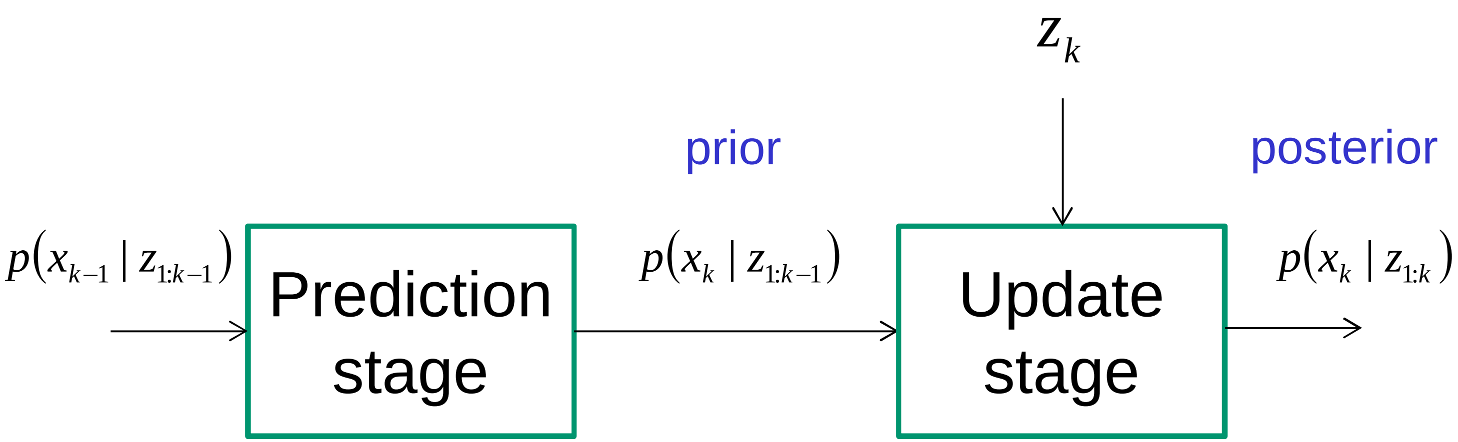


- Update stage: Bayes Rule

posterior, Likelihood, prior

$$p(x_k \mid z_{1:k}) = \frac{p(z_k \mid x_k)\, p(x_k \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$$

From the normalization

$$\int p(x_k \mid z_{1:k})\, dx_k = 1 \quad \longrightarrow \quad p(z_k \mid z_{1:k-1}) = \int p(z_k \mid x_k) \cdot p(x_k \mid z_{1.k-1})\, dx_k$$

The sequential solution: What is difficult in practice?

1) The probability distributions are not usually available in close form!

$$p(x_{k-1} \mid z_{1:k-1})$$

2) The integrals are difficult to solve analytically!

$$p(x_{0:k} \mid z_{1:k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid z_{1:k-1}) dx_{k-1}$$

$$p(x_k \mid z_{1:k}) = \frac{p(z_k \mid x_k) p(x_k \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$$

$$p(z_k \mid z_{1:k-1}) = \int p(z_k \mid x_k) \cdot p(x_k \mid z_{1.k-1}) dx_k$$

Available model-based filtering techniques

Kalman Filter	Extended-Kalman Filter	Approximate Grid-based filters
Exact only for linear systems and additive Gaussian noises	Analytical approximation	Numerical approximation (burdensome)

PARTICLE FILTERING

Numerical solution which, in the limit, tends to the exact posterior pdf:

$$p\left(x_k \mid z_{1:k}\right)$$

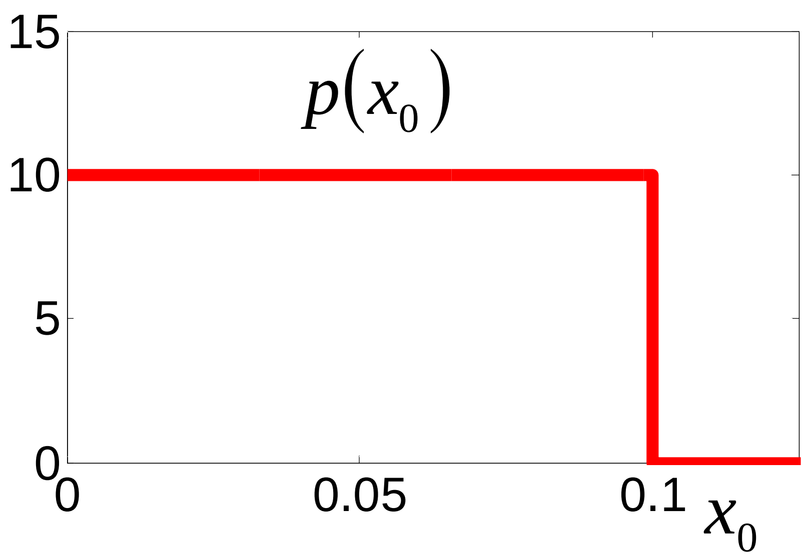
- Particle filtering for degradation state estimate
  - The intuitive representation
  - Detailed analytical approach to the problem
  - The pseudocode
  - State estimate in practice

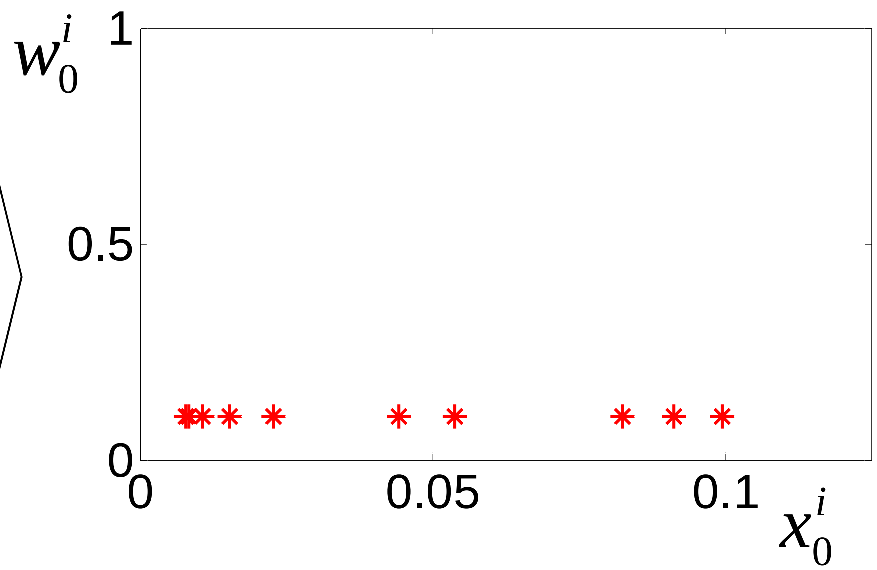
The intuitive representation:
1. pdf approximation

Å Time 0, we approximate $p(x_0)$ in the form of a set of N_s random samples x_0^i with associated weights $w_0^i = \frac{1}{N_s}$: $\{x_0^i, w_0^i\}$

$$p(x_0) \approx \sum_{i=1}^{N_s} w_0^i \delta(x_0 - x_0^i)$$

$$\sum_{i=1}^{N_s} w_0^i = 1$$





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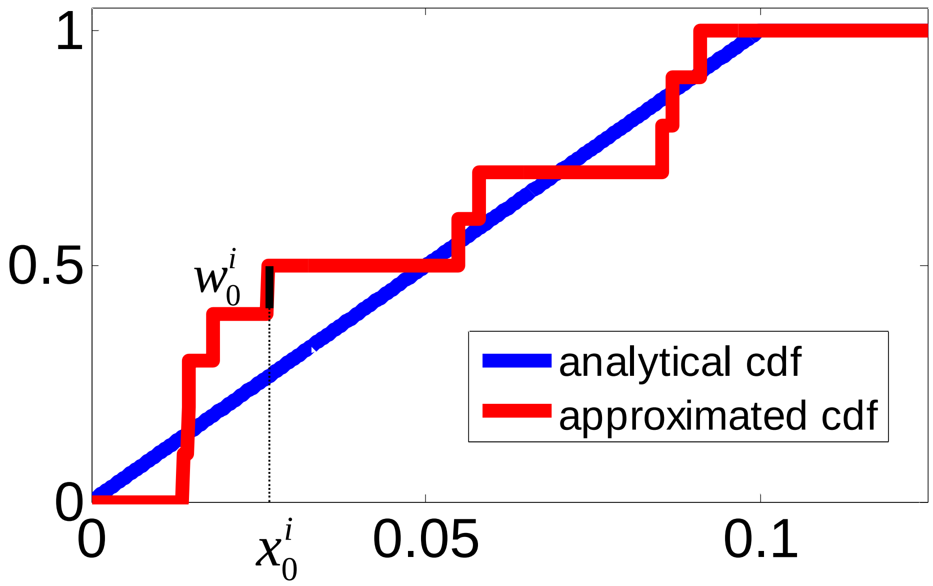
$$p(x_0) \approx \sum_{i=1}^{N_s} w_0^i \delta(x_0 - x_0^i)$$

Cumulative distribution:

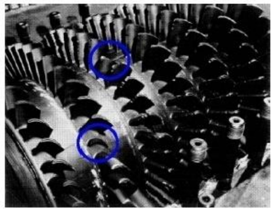
$$\int_0^{x_0} p(x')dx'$$



$p(x_0)$ → **Prediction stage** → $p(x_1)$

$\{x_0^i, w_0^i\}$ $\qquad$ $\{x_1^i = f(x_0^i, \omega_0^i), w_0^i\}$

$$x_1 = f(x_0, \omega_0)$$



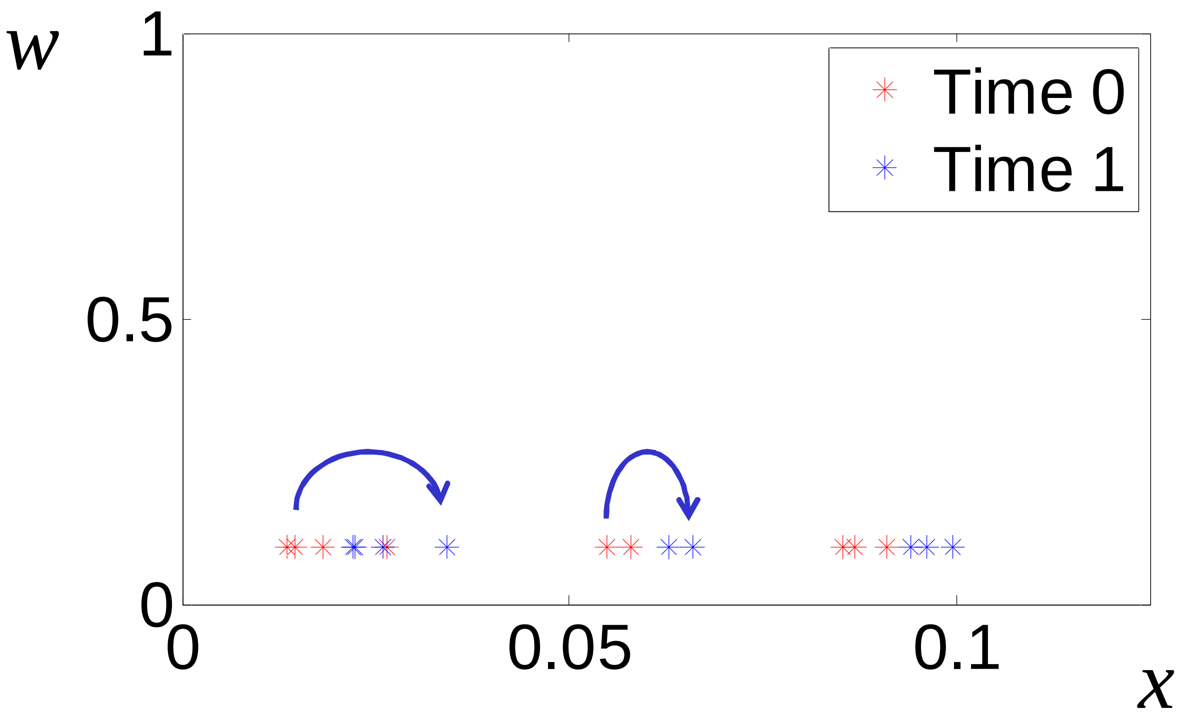
Prediction stage for particle *i*

1. Sample a value of $\omega_1^i, \omega_2^i, \omega_3^i$
2. Apply:

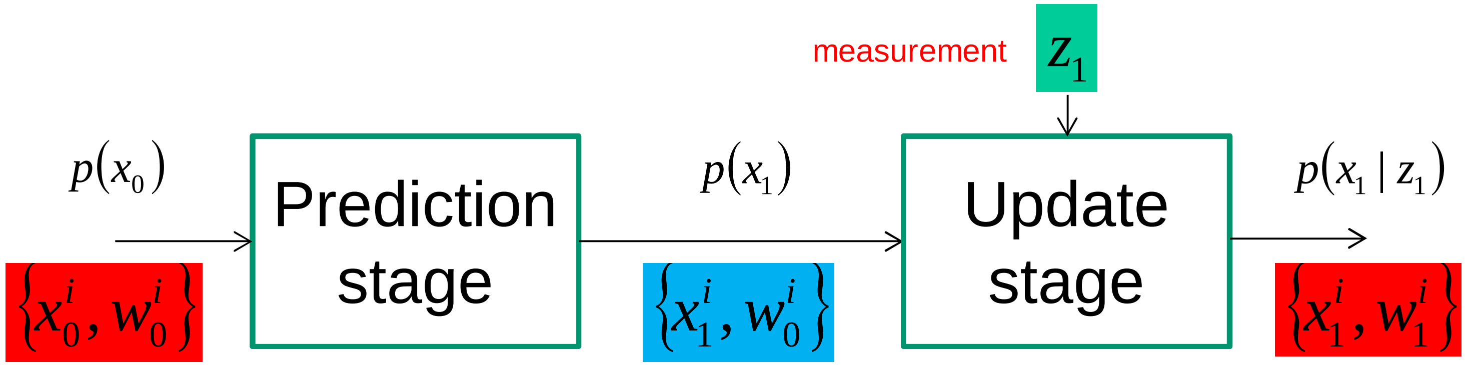
$$x_1^i = x_0^i + A\exp\left(-\frac{Q}{R\left(T_0 + \omega_1^i\right)}\right)\left(K\left(\theta_0 + \omega_2^i\right)^2 + \omega_3^i\right)^n$$

The intuitive representation: prediction stage: Monte Carlo Simulation

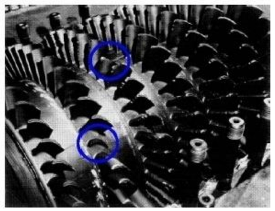
$$x_1^i = x_0^i + A\exp\left(-\frac{Q}{R\left(T_0+\omega_1^i\right)}\right)\left(K\left(\theta_0+\omega_2^i\right)^2+\omega_3^i\right)^n$$

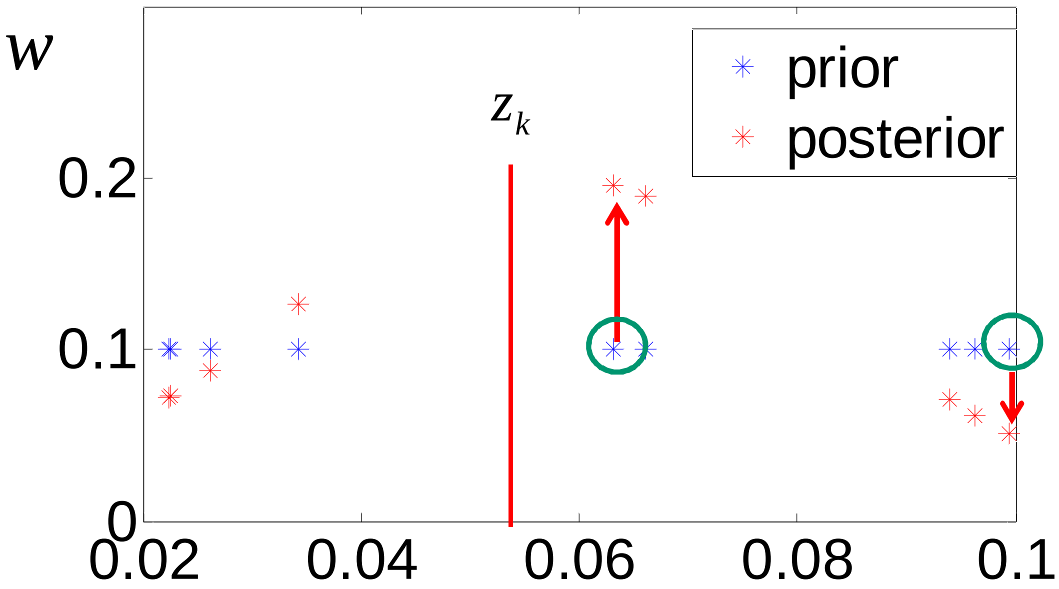


The intuitive representation: update stage: weight modification

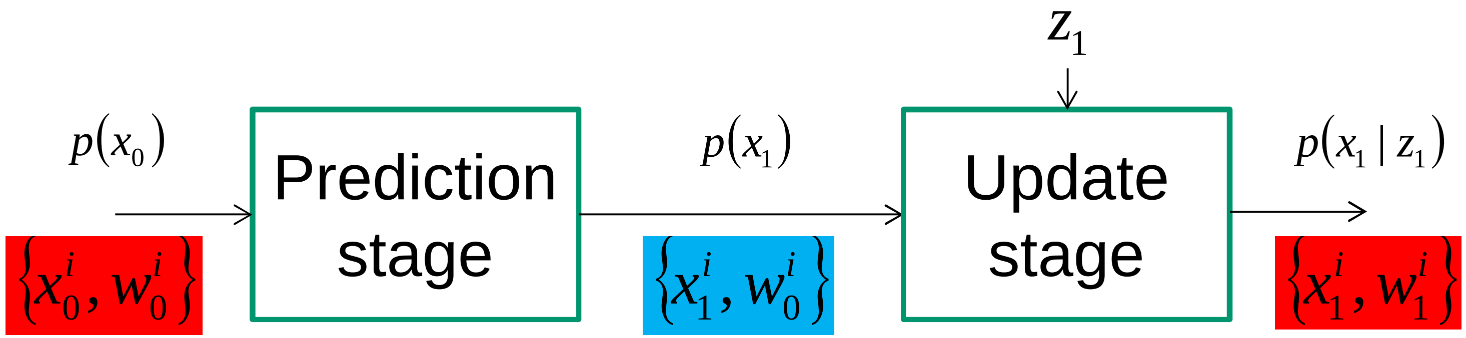


Å Time 1: measure $z_1 = 0.058$ becomes available → particle weightsôupdate



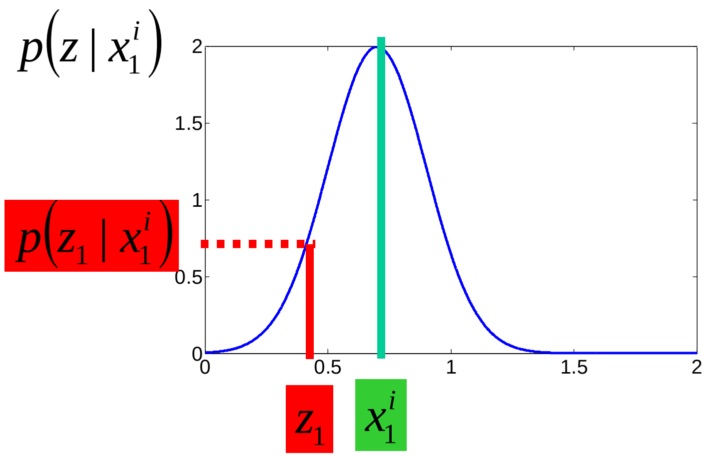


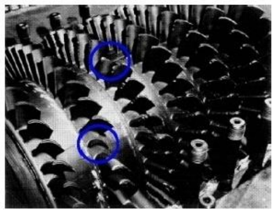
The intuitive representation: update stage: weight modification



- Time 1: measure z_1 becomes available

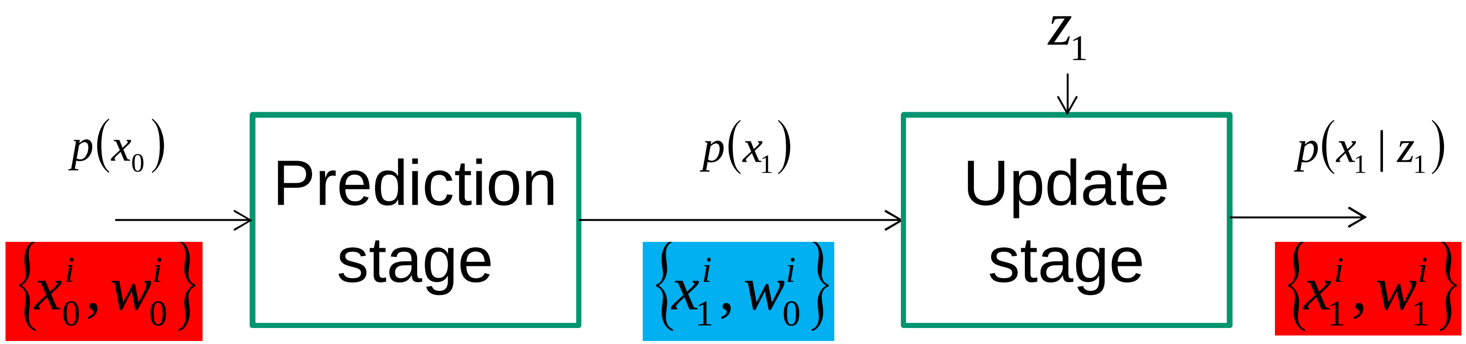
- Compute likelihood of the particles: $p(z_1 | x_1^i)$





$$z = x + \nu$$
$$\nu \propto N(0, \sigma^2)$$

The intuitive representation: update stage: weight modification



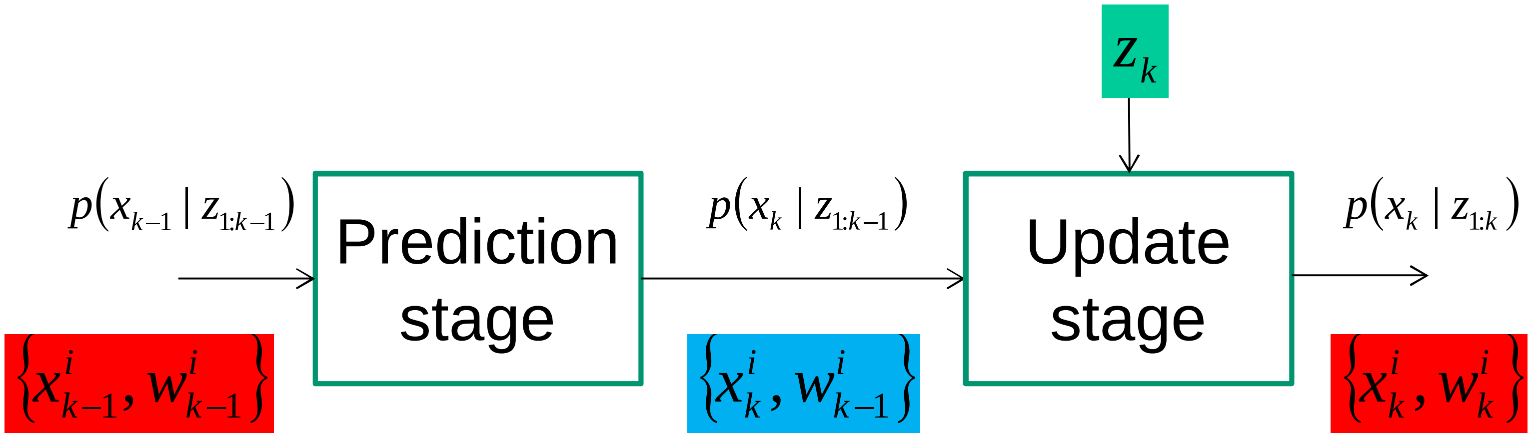
Å Time 1: measure z_1 becomes available

Å Compute likelihood of the particles: $p(z_1 | x_1^i)$

Å $w_1 = w_0 \cdot p(z_1 | x_1^i)$

The intuitive representation

Å Repeat prediction and update stage each time a new measure becomes available

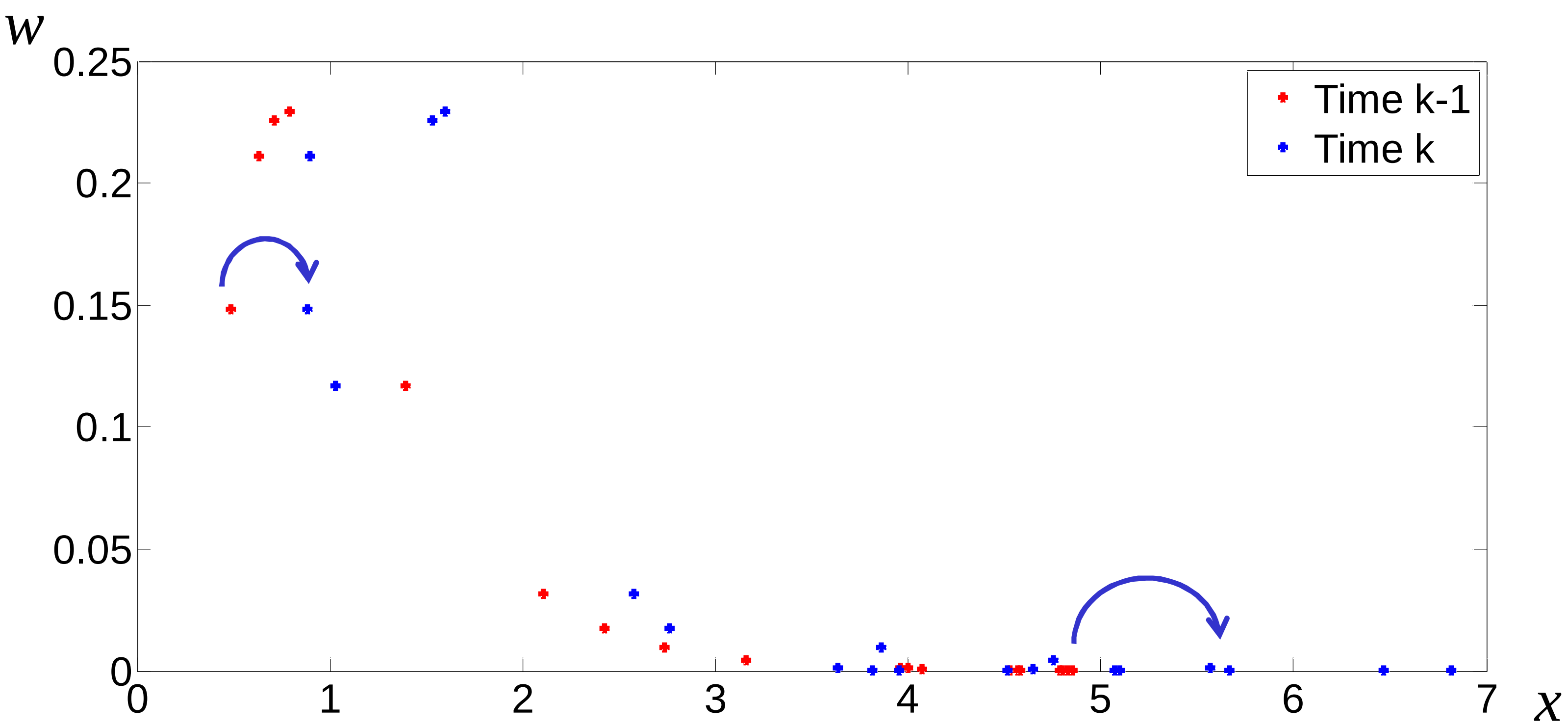


The intuitive representation: prediction stage: Monte Carlo Simulation

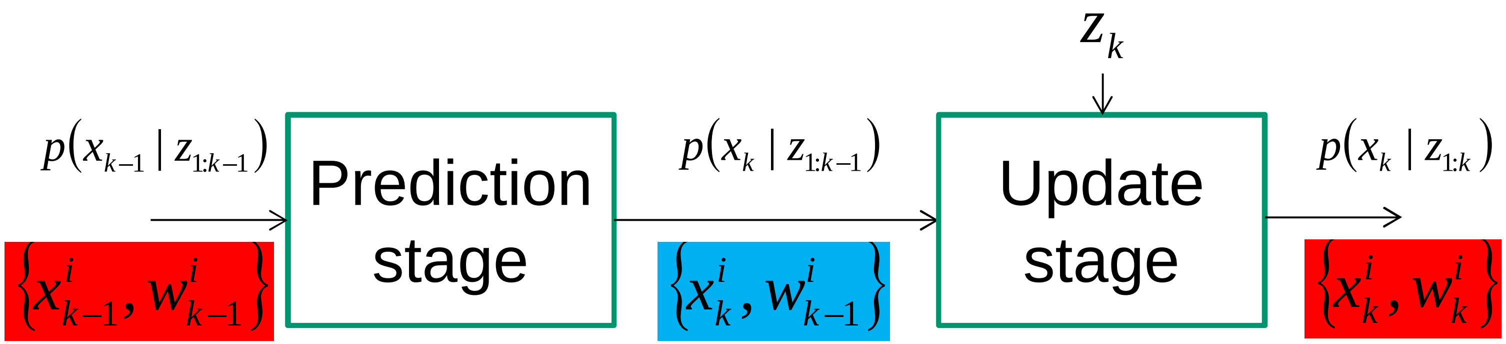
$p(x_{k-1} \mid z_{1:k-1})$ → **Prediction stage** → $p(x_k \mid z_{1:k-1})$

$\{x_{k-1}^i, w_{k-1}^i\}$

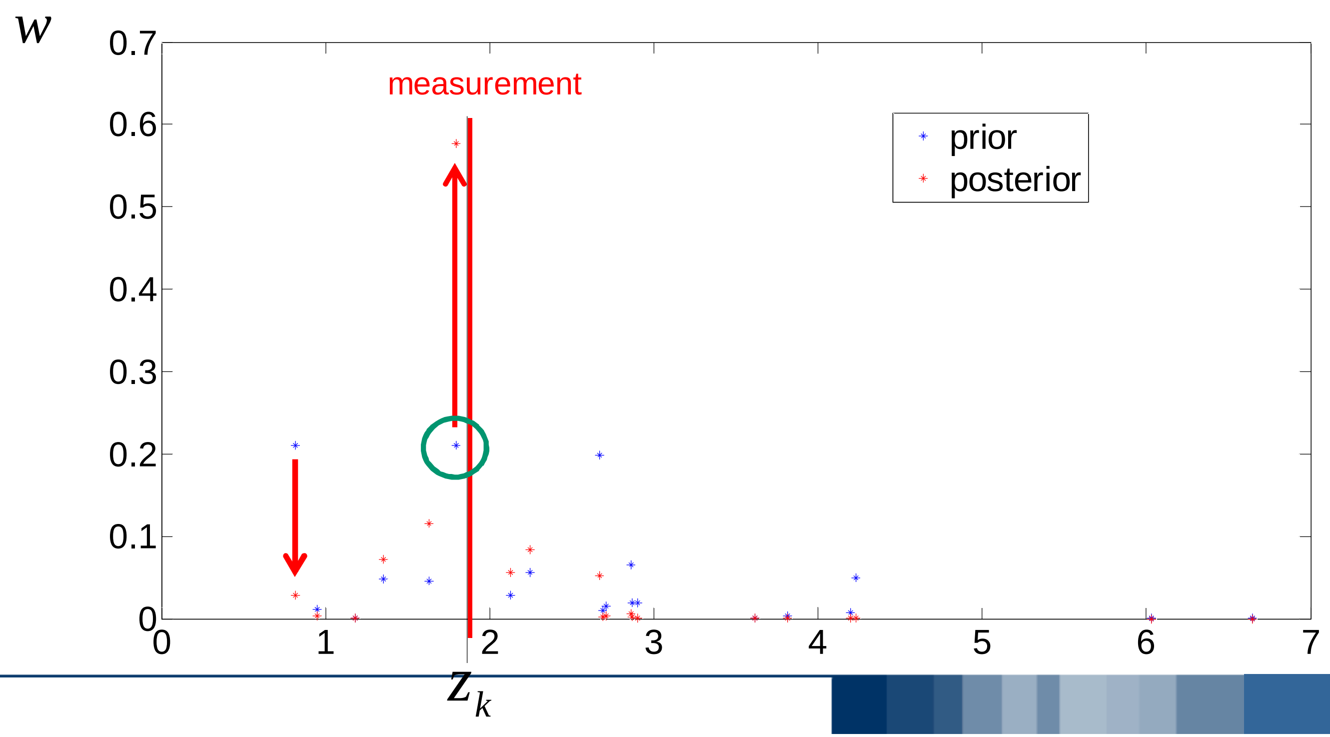
$\{x_k^i = f_k(x_{k-1}^i, \omega_{k-1}^i), w_{k-1}^i\}$



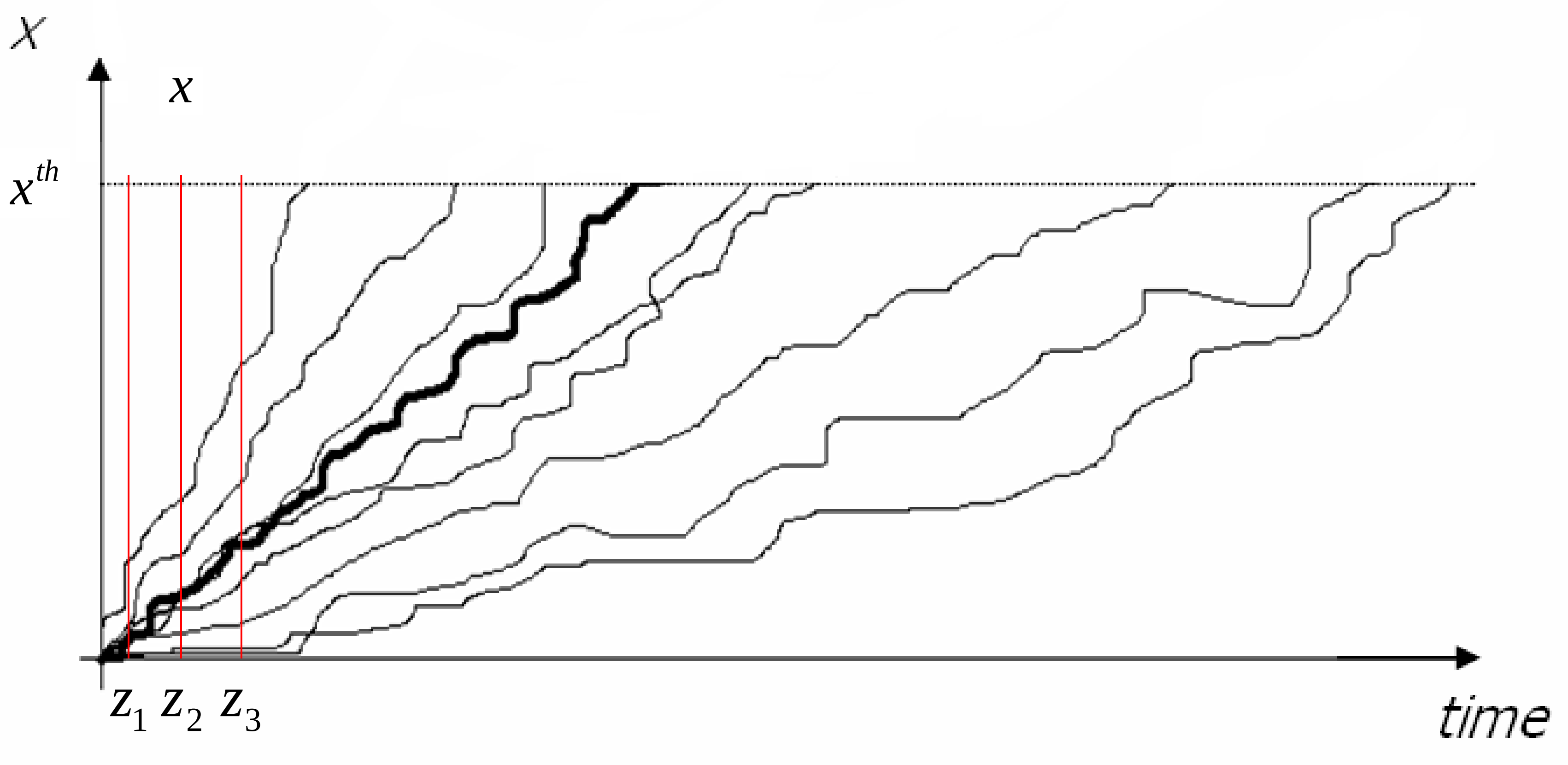
The intuitive representation:
update stage: weight modification



- Time k: measure z_k becomes available → particle weight modification



Example of Particle Trajectories



Estimates of interest

$$\left\{x_k^i, w_k^i\right\}$$

$$p(x_k \mid z_{1:k}) \approx \sum_{i=1}^{N_s} \tilde{w}_k^i \, \delta\left(x_k - x_k^i\right)$$

- degradation state **mean** (estimate) $$\hat{x}_k = \sum_{i=1}^{N_s} w_k^i x_k^i$$

- degradation state **variance** (uncertainty) $$\hat{\sigma}_k^2 = \sum_{i=1}^{N_s} w_k^i \left(x_k^i - \hat{x}_k\right)^2$$

- Particle filtering for degradation state estimate
  - The intuitive representation
  - Detailed analytical approach to the problem
  - The algorithm
  - State estimate in practice

Basic Idea: Importance sampling

OBJECTIVE: $p\left(x_{0:k} \mid z_{1:k}\right)$

MAIN IDEA: IMPORTANCE SAMPLING

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

Importance sampling

Å Let $p(x) \propto \pi(x)$ be a probability density function (pdf) difficult to sample from, with $\pi(x)$ easy to evaluate

Å Let $q(x)$ be a proposal pdf easy to sample from: $\{x^i\}_{i=1:N_S}$

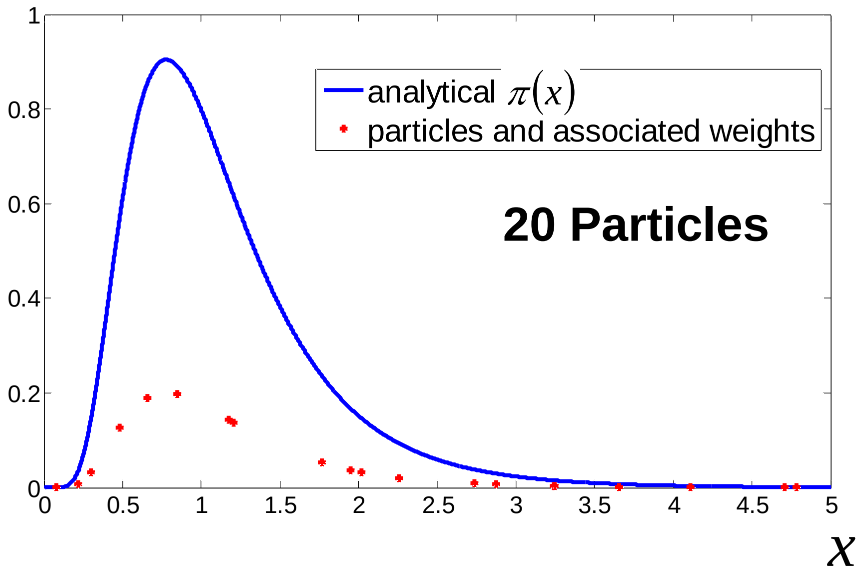
Importance density

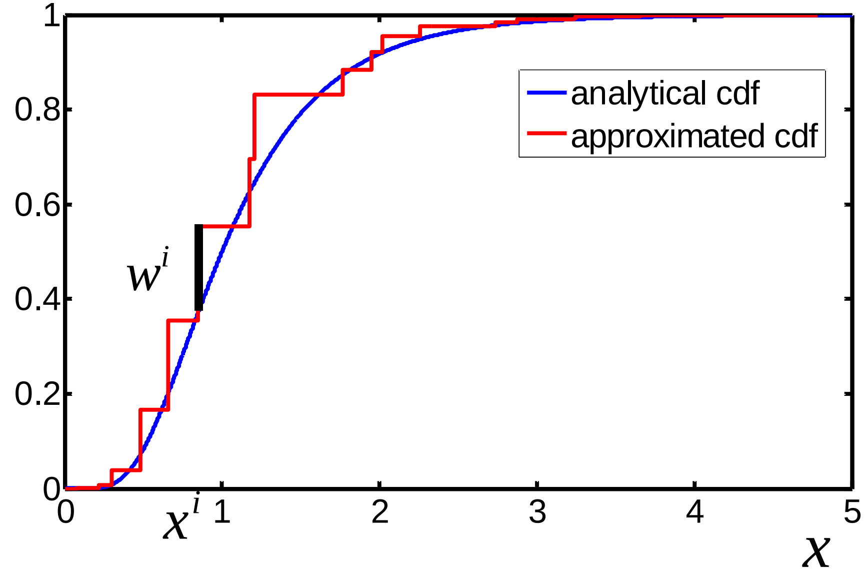
$$p(x) \approx \sum_{i=1}^{N_s} w^i \delta(x - x^i)$$

where:

$$\tilde{w}^i = \frac{\pi(x^i)}{q(x^i)} \qquad w^i = \frac{\tilde{w}^i}{\sum_{i=1,N_S} \tilde{w}^i}$$

Example: approximation of the pdf distribution

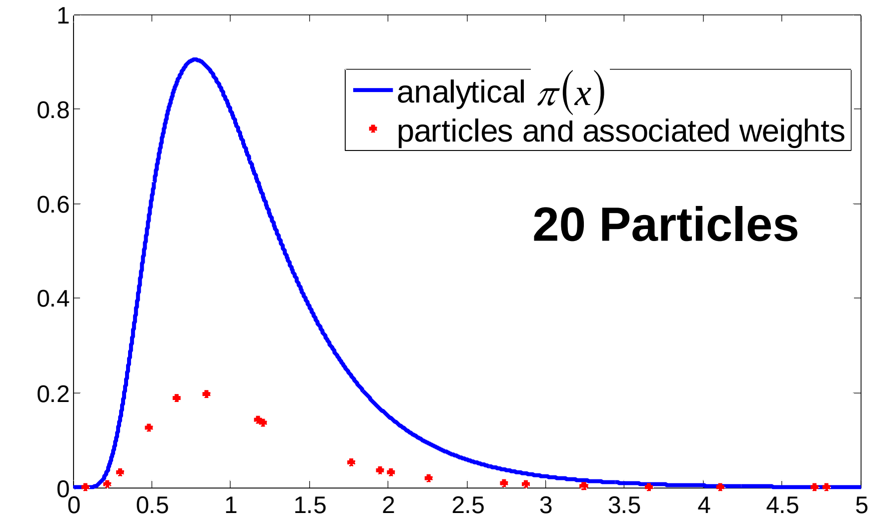


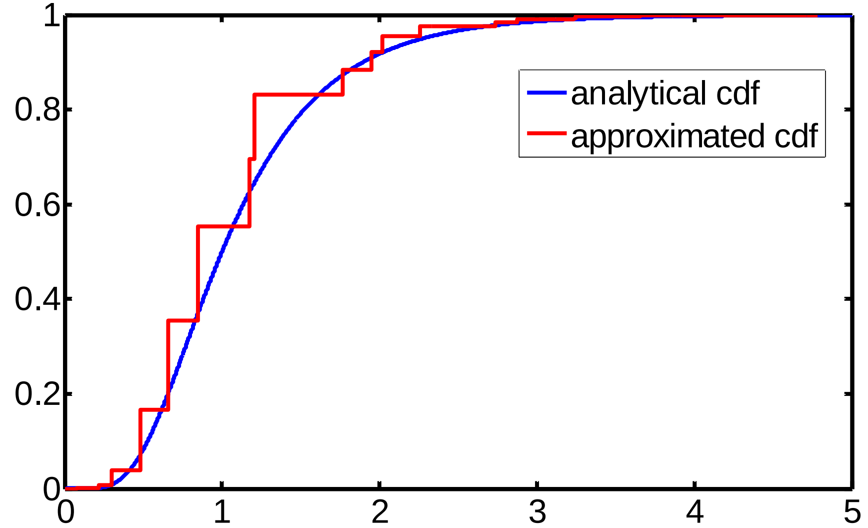


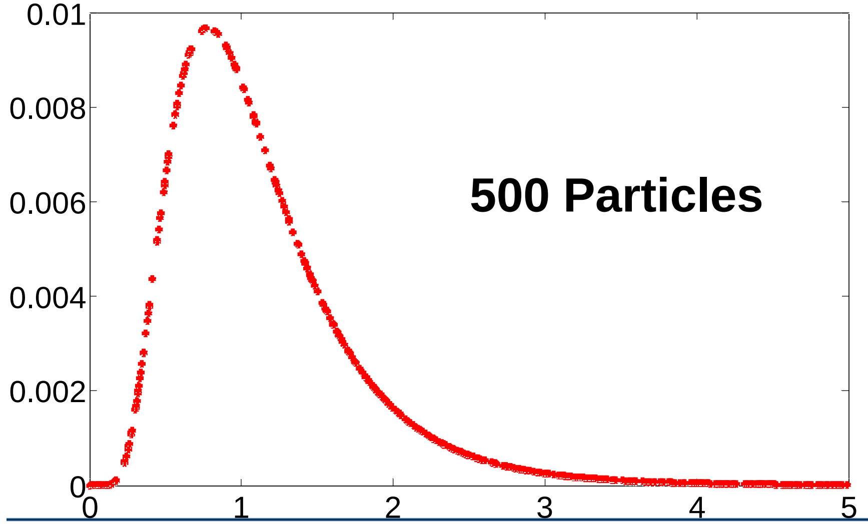
- Particles sampled from: $q(x)=U[0,5]$
- Corresponding weight obtained from:

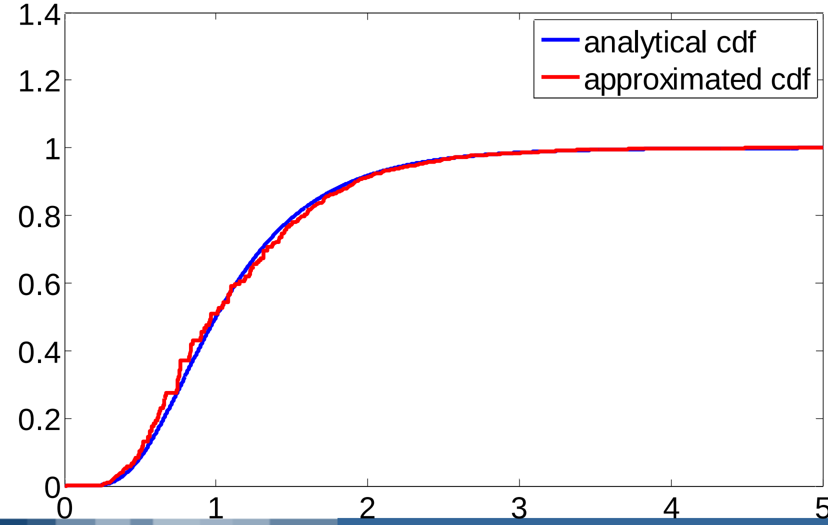
$$\tilde{w}^i = \frac{\pi(x^i)}{q(x^i)} = \frac{\pi(x^i)}{1/5}$$

Example: approximation of the pdf distribution









Particle Filter: Estimate of the posterior

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

Arbitrarily chosen

In practice:

Å Sample N_s particles from $q\left(x_{0:k} \mid z_{1:k}\right)$

Å Compute weights from:

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)}$$

Sequential Importance Sampling

Arbitrarily chosen

$$q(x_{0:k} \mid z_{1:k}) = q(x_k \mid x_{0:k-1}, z_{1:k})\, q(x_{0:k-1} \mid z_{1:k-1})$$

Known from previous time step

Sample at time *k*-1: $x_0^i, x_1^i, \ldots, x_{k-1}^i$

Sample at time *k*: $x_0^i, x_1^i, \ldots, x_{k-1}^i,$

from $q(x_k \mid x_{0:k-1}, z_{1:k})$

Particle Filter: Estimate of the posterior

$$p\left(x_{0:k} \mid z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{0:k} - x_{0:k}^i\right)$$

In practice:

Å Sample N_s particles from $q\left(x_{0:k} \mid z_{1:k}\right) = q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) q\left(x_{0:k-1} \mid z_{1:k-1}\right)$

Å Compute weights from:

?

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)} = \frac{\boxed{p\left(x_{0:k}^i \mid z_{1:k}\right)}}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right) q\left(x_{0:k-1}^i \mid z_{1:k-1}\right)}$$

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$p\left(x_{0:k} \mid z_{1:k}\right) = \frac{p\left(x_{0:k} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

Å Bayes Rule

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

(conditional probability formula)

$$P(A,B) = P(A \mid B)P(B)$$

$$p\left(x_k, x_{0:k-1} \mid z_{1:k-1}\right) = p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)$$

$$p\left(x_{0:k} \mid z_{1:k}\right) = \frac{\cdots\, p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

$$= \frac{\cdots\, p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$
\begin{aligned}
p\left(x_{0:k} \mid z_{1:k}\right) &= \frac{p\left(x_{0:k} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= \frac{\boxed{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)} p\left(x_{0:k-1} \mid z_{1:k-1}\right) \boxed{p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= \frac{\boxed{p\left(x_k \mid x_{0:k-1}\right)} p\left(x_{0:k-1} \mid z_{1:k-1}\right) \boxed{p\left(z_k \mid x_{0:k}\right)}}{p\left(z_k \mid z_{1:k-1}\right)}
\end{aligned}
$$

(observational independence)

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$p\left(x_{0:k} \mid z_{1:k}\right) = \frac{p\left(x_{0:k} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

$$= \frac{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right) p\left(x_{0:k-1} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

$$= \frac{p\left(x_k \mid x_{0:k-1}\right) p\left(x_{0:k-1} \mid z_{1:k-1}\right) p\left(z_k \mid x_{0:k}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

$$= p\left(x_{0:k-1} \mid z_{1:k-1}\right) \frac{p\left(z_k \mid x_{0:k}\right) p\left(x_k \mid x_{0:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}$$

Rearrangement

Recursive formula for $p\left(x_{0:k}^{i} \mid z_{1:k}\right)$

$$
\begin{aligned}
p\left(x_{0:k} \mid z_{1:k}\right) &= \frac{p\left(x_{0:k} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= \frac{p\left(x_k \mid x_{0:k-1}, z_{1:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}, z_{1:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= \frac{p\left(x_k \mid x_{0:k-1}\right)p\left(x_{0:k-1} \mid z_{1:k-1}\right)p\left(z_k \mid x_{0:k}\right)}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= p\left(x_{0:k-1} \mid z_{1:k-1}\right)\frac{p\left(z_k \mid x_{0:k}\right)p\left(x_k \mid x_{0:k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)} \\
&= p\left(x_{0:k-1} \mid z_{1:k-1}\right)\frac{p\left(z_k \mid x_k\right)p\left(x_k \mid x_{k-1}\right)}{p\left(z_k \mid z_{1:k-1}\right)}
\end{aligned}
$$

(Markov model)

$$\propto p\left(z_k \mid x_k\right)p\left(x_k \mid x_{k-1}\right)$$

Weight updating equation ï Sequential Importance Sampling (SIS)

Å Where were we?

- SLIDE 62:

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)}$$

$$= q\left(x_k \mid x_{0:k-1}, z_{1:k}\right)$$

- SLIDE 67:

$$\propto p\left(z_k \mid x_k\right) p\left(x_k \mid x_{k-1}\right)$$

$$w_k^i \propto \frac{p\left(x_{0:k}^i \mid z_{1:k}\right)}{q\left(x_{0:k}^i \mid z_{1:k}\right)} \propto \frac{p\left(z_k \mid x_k^i\right) p\left(x_k^i \mid x_{k-1}^i\right) p\left(x_{0:k-1}^i \mid z_{1:k-1}\right)}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right) q\left(x_{0:k-1}^i \mid z_{1:k-1}\right)} = \frac{p\left(z_k \mid x_k^i\right) p\left(x_k^i \mid x_{k-1}^i\right)}{q\left(x_k^i \mid x_{0:k-1}^i, z_{1:k}\right)} w_{k-1}^i$$

A possible choice for $q\left(x_k \mid x_{0:k-1}, z_{1:k}\right)$

MOST POPULAR CHOICE

$$q\left(x_k \mid x_{0:k-1}, z_{1:k}\right) = p\left(x_k \mid x_{k-1}\right)$$

Easy! We know the Physical model of the degradation process

$$\tilde{w}_k^i = w_{k-1}^i \frac{p\left(z_k \mid x_k^i\right) p\left(x_k^i \mid x_{k-1}^i\right)}{q\left(x_k^i \mid x_{0:k-1}^i, z_k\right)} = w_{k-1}^i p\left(z_k \mid x_k^i\right)$$

Easy! We know the measurement equation

Advantage:

- easy to implement (both sampling and evaluation of weights)

Drawbacks:

- state-space explored without knowledge of observations
- degeneracy phenomenon

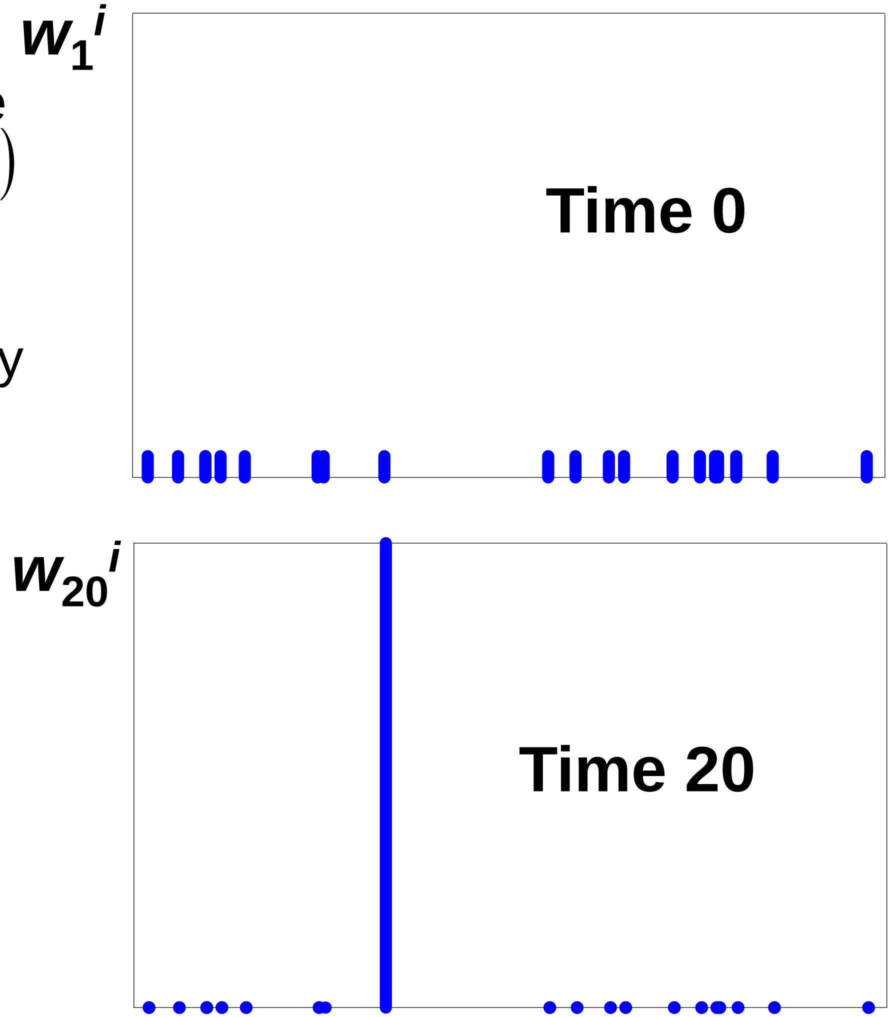
SIS: degeneracy problem

Variance of the weights can only increase over time: $w_k^i = w_{k-1}^i p\left(z_k \mid x_k^i\right)$

Weight distribution becomes progressively more skewed

Large effort in updating particles whose contribution to final estimate is almost 0

Resampling Algorithm



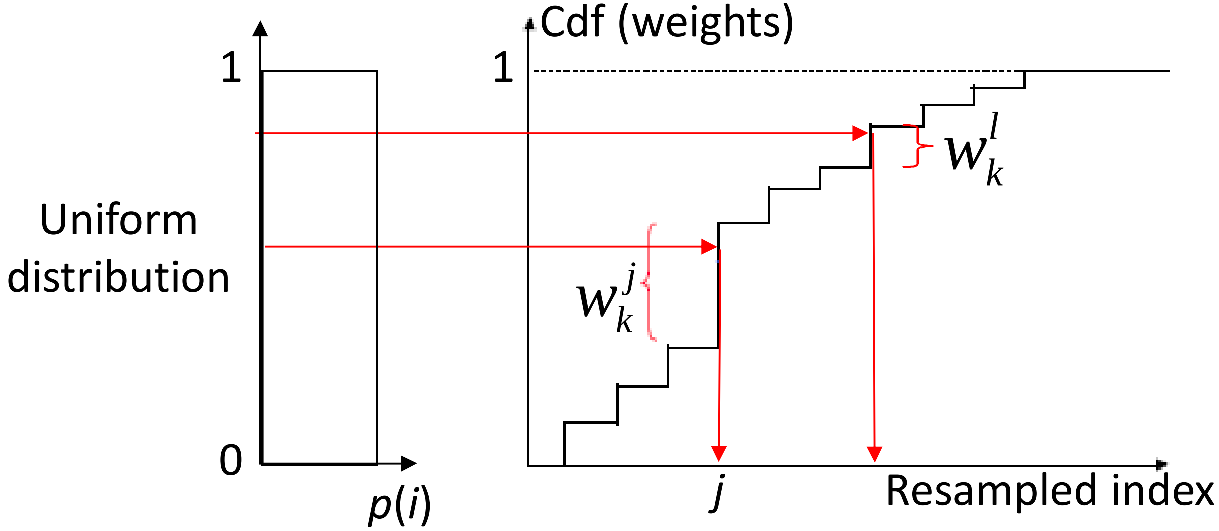
Bootstrap resampling procedure

Å Reduce number of samples with low weights and increase number of samples with large weights

Å Set of unequally weighted samples Ÿ set of equally weighted particles

$$\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s} \rightarrow \left\{x_k^{j*}, 1/N_s\right\}_{j=1}^{N_s}$$

BOOTSTRAP RESAMPLING WITH REPLACEMENT



$$p\left(x_k^{j*} = x_k^i\right) = w_k^i$$

- Particle filtering for degradation state estimate
  - The intuitive representation
  - Detailed analytical approach to the problem
  - The pseudo-code
  - State estimate in practice

Sampling Importance Resampling (SIR) PF

$$\left[\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s}\right] = \mathrm{SIR\text{-}PF}\left[\left\{x_{k-1}^i, w_{k-1}^i\right\}_{i=1}^{N_s}, z_k\right]$$

For i = 1: N_s

- Sample: x_k^i using x_{k-1}^i and $x_k = f_k\left(x_{k-1}, \omega_{k-1}\right)$

- Assign the particles a weight: $\tilde{w}_k^i = w_{k-1}^i p\left(z_k \mid x_k^i\right)$

End For

For i = 1: N_s

- Normalize the weights: $w_k^i = \tilde{w}_k^i \Big/ \sum_{i=1}^{N_s} \tilde{w}_k^i$

End For

é

Sampling Importance Resampling (SIR) PF

é

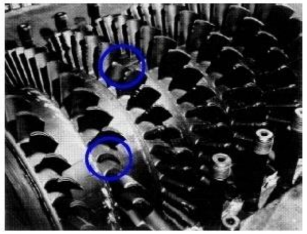
- $\left[\left\{x_k^{j*}, w_k^{j*} = 1/N_s\right\}_{i=1}^{N_s}\right]$ = RESAMPLE $\left[\left\{x_k^i, w_k^i\right\}_{i=1}^{N_s}\right]$
    - Bootstrap sample the system states (with replacement)
    - Update the weights: $w_k^{j*} = 1/N_s$

- Compute estimates of interest:
    - Posterior mean: $\hat{x}_k = \sum_{i=1}^{N_s} w_k^i x_k^i$
    - Posterior variance: $\hat{\sigma}_k^2 = \sum_{i=1}^{N_s} w_k^i \left(x_k^i - \hat{x}_k\right)^2$

End SIR-PF

- Particle filtering for degradation state estimate
  - The intuitive representation
  - Detailed analytical approach to the problem
  - The algorithm
  - State estimate in practice

Degradation state estimate in practice

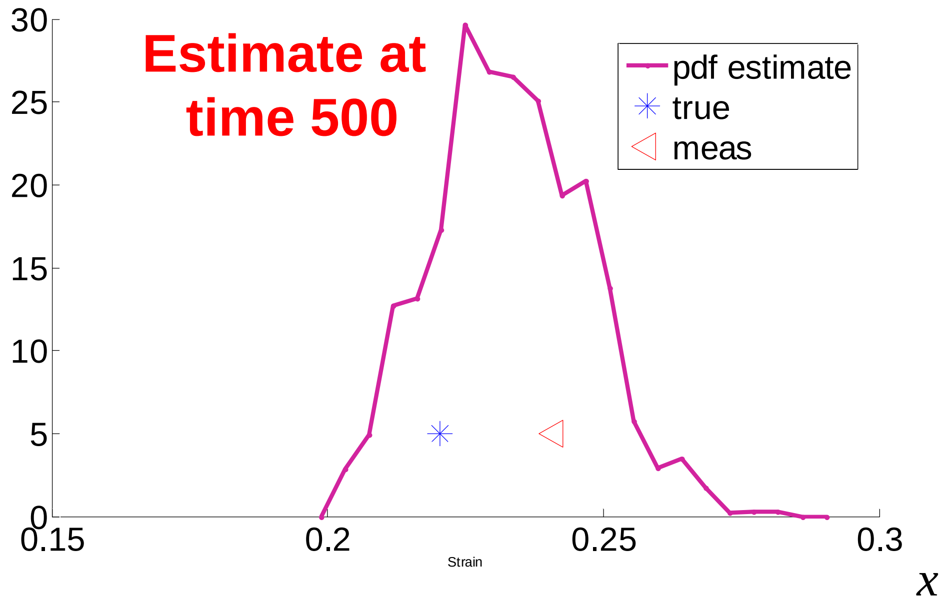


$$x_k = x_{k-1} + A\exp\left(-\frac{Q}{R(T_0+\omega_1)} \right)\left(K(\theta_0+\omega_2)^2 \right)^n$$

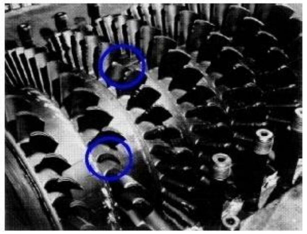
n=6
A= 7.5e^{-3} %/(MPa^n*day)
Q: Activation energy = 290000 J/mol
R: Ideal gas constant = 8.31 J/(mol*K)
K=0. 0011 MPa
T_0 = 1100 K
θ_0 = 3000 rpm
ω_1~ N(0; 11) K
ω_2~ N(0; 30) rpm

Initial Condition: Time t=0 → $x_0 = 0$
Number of Particles: N_p =1000

Time	Elongation Measure
500	0.2411%



Degradation state estimate in practice

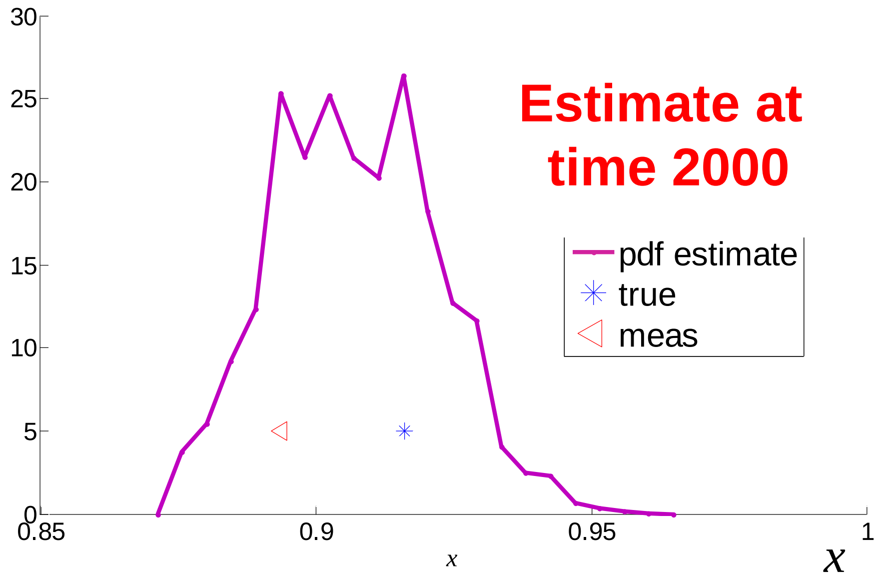


$$x_k = x_{k-1} + A\exp\left(-\frac{Q}{R(T_0+\omega_1)}\right)\left(K(\theta_0+\omega_2)^2\right)^n$$

n=6
A= 7.5e^{-3} %/(MPa^n*day)
Q: Activation energy = 290000 J/mol
R: Ideal gas constant = 8.31 J/(mol*K)
K=0. 0011 MPa
T_0 = 1100 K
θ_0 = 3000 rpm
ω_1~ N(0; 11) K
ω_2~ N(0; 30) rpm

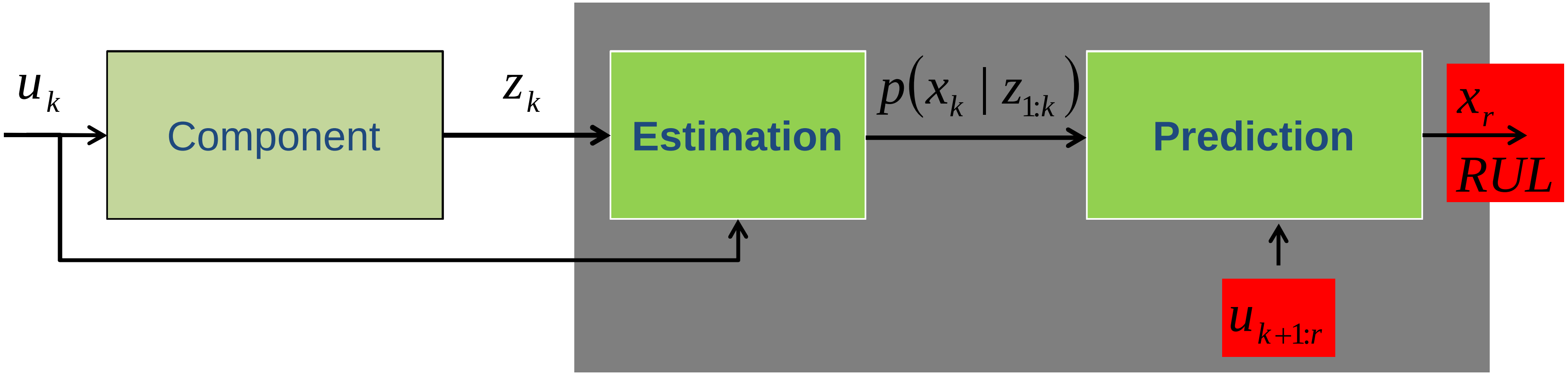
Initial Condition: Time t=0 → $x_0 = 0$
Number of Particles: N_p =1000

Time	Elongation Measure
500	0.2411%
1000	0,4600%
1500	0,7129
2000	0,8938



- Model-based prognostics:
  - The filtering problem
  - The forecasting problem

The forecasting problem



Information Available:

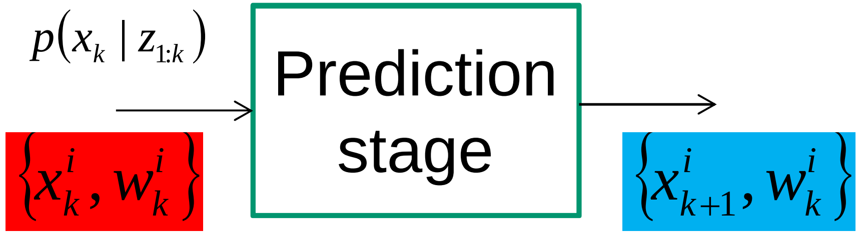
- Estimate of the pdf of the state at the current time (from PF): $p(x_k \mid z_{1:k})$ in the form of $\{x_k^i, w_k^i\}_{i=1}^{N_s}$
- future (random) distribution of the operational/external conditions: $p_r(u_r, \omega_r)$
- physical model of the degradation process $x_k = f_k(x_{k-1}, \omega_{k-1})$

- Estimate $p(x_r \mid z_{1:k})$
- Estimate RUL

- The forecasting problem
  - Particle Filtering for RUL estimate

RUL estimate: Method

Å Prediction of the degradation state one time step ahead:



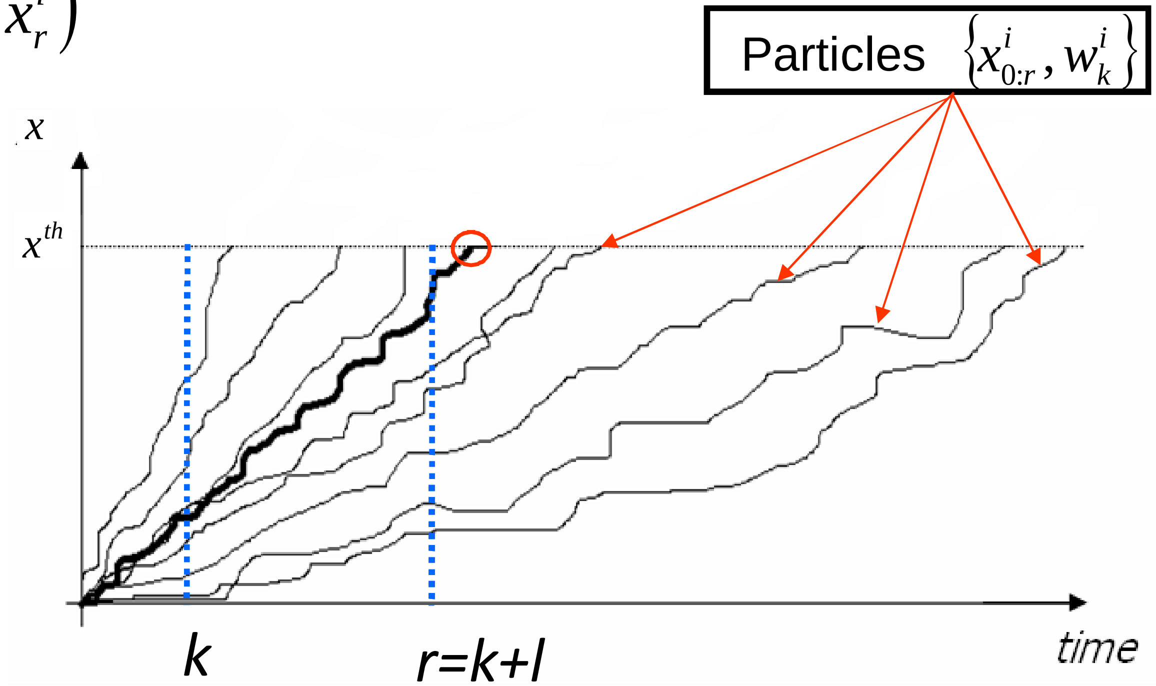
$$x_{k+1}^i = f_k\left(x_k^i, \omega_k^i\right)$$

$$p\left(x_{k+1} | z_{1:k}\right) \approx \sum_{i=1}^{N_s} w_k^i \delta\left(x_{k+1} - x_{k+1}^i\right)$$

RUL estimate: Method

- Prediction stage at *r-k* time step ahead:

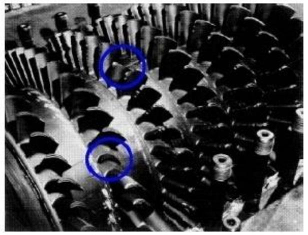
$$p(x_r \mid z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(x_r - x_r^i)$$



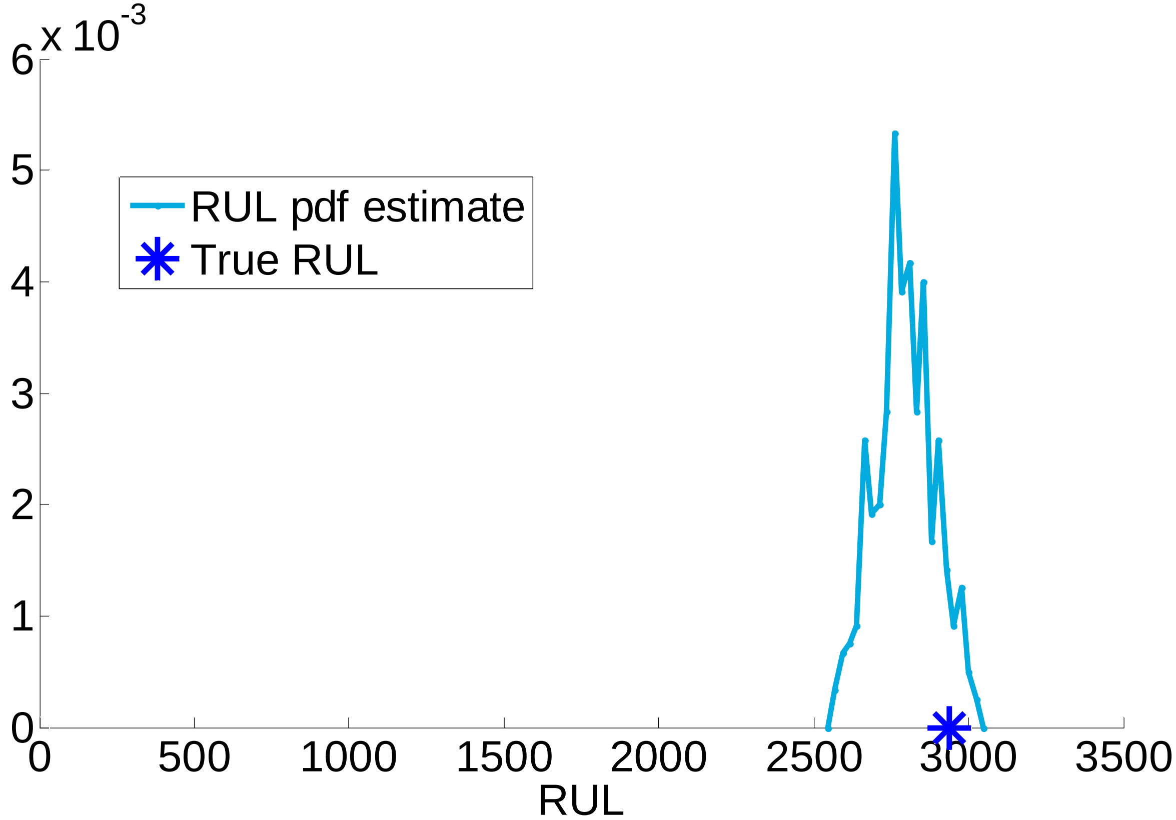
- RUL estimate

$$p(rul \mid z_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(rul - rul^i)$$

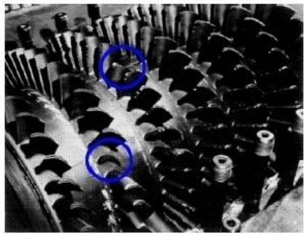
RUL estimate in practice



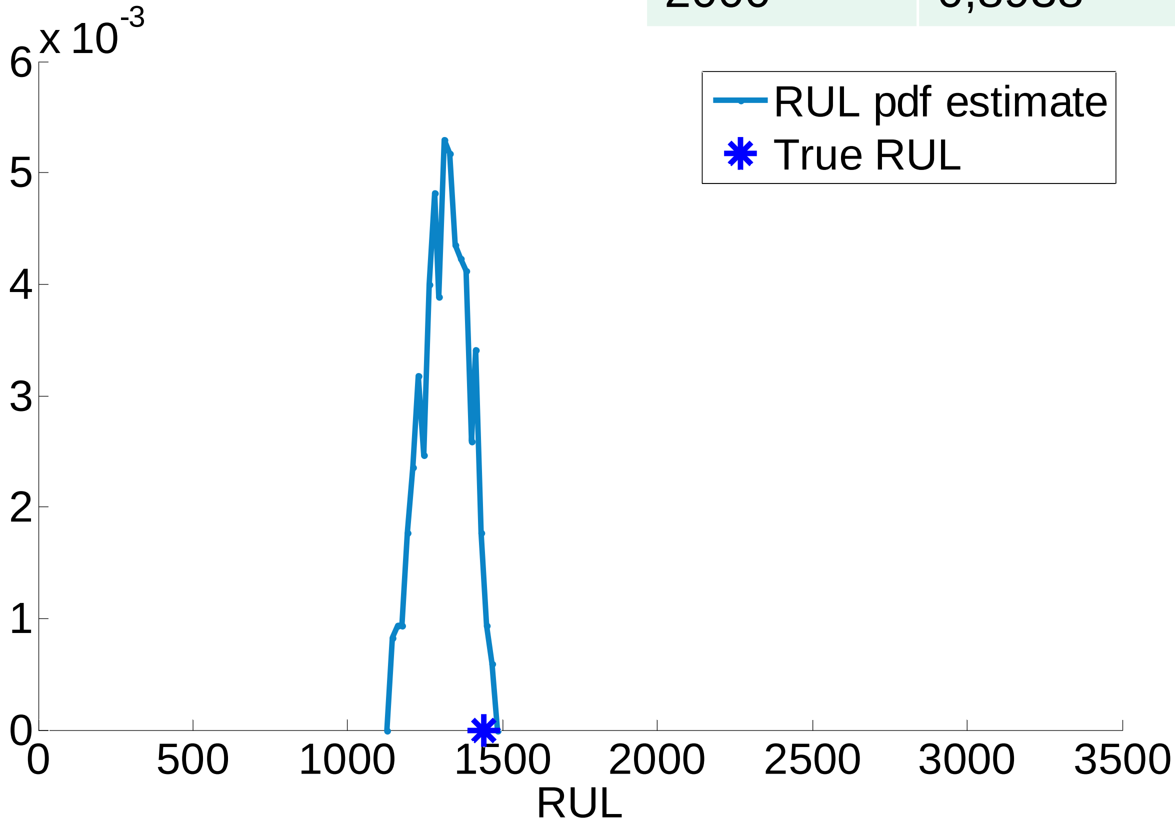
Time	Elongation Measure
500	0.2411%



RUL estimate in practice

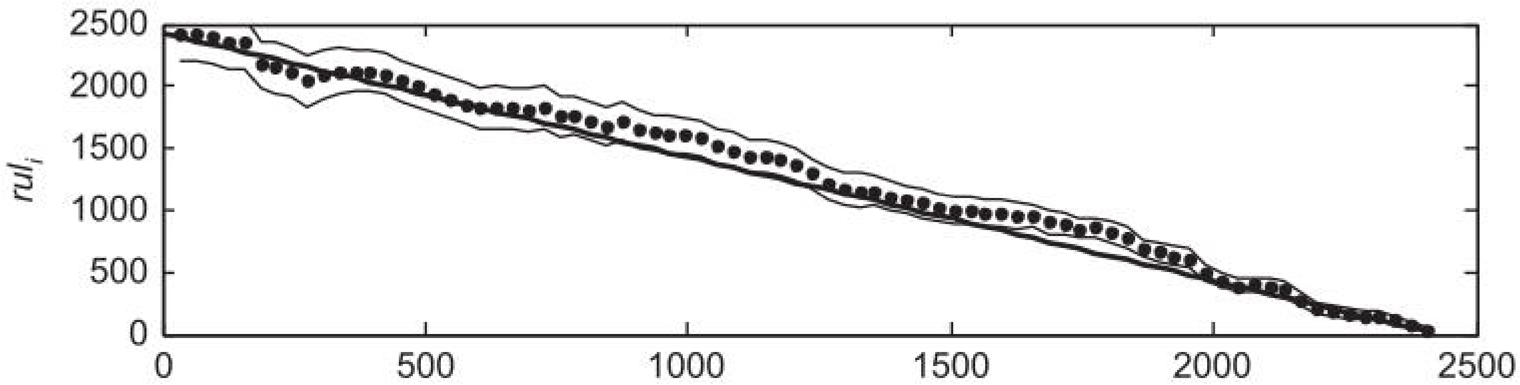


Time	Elongation Measure
500	0.2411%
1000	0,4600%
1500	0,7129
2000	0,8938



RUL estimate in practice: performance

Å Another test case: one creep elongation measure every month



Å Test over N_{tst} = 250 different creep growth trajectories

Å Mean Relative Absolute Error:

$$rMAE = \frac{1}{N_{tst}} \sum_{i=1}^{N_{tst}} \left| \frac{rul_i - \hat{rul}_i}{rul_i} \right| = 0.150 \pm 0.009$$

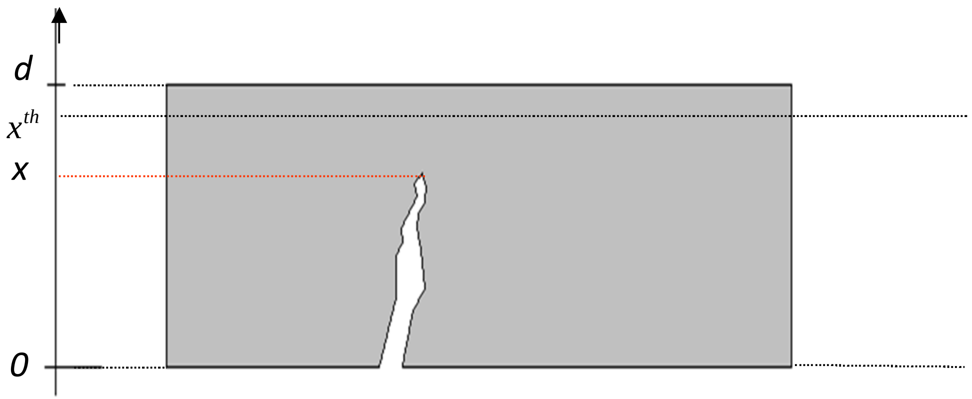
Å Coverage:

$$Cov = \frac{1}{N_{tst}} \sum_{i=1}^{N_{tst}} c_i; \quad c_i = \begin{cases} 1 & if \quad rul_i \in C_i^{68\%} \\ 0 & if \quad rul_i \notin C_i^{68\%} \end{cases} \qquad Cov = 0.663 \pm 0.018$$

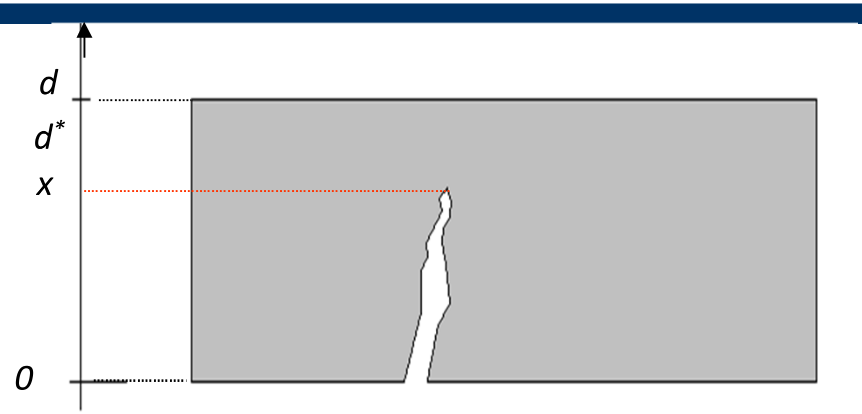
- Application:
  - Maintenance Planning for a degrading structure

The degrading component

Component: structure
Degradation mechanism: crack propagation
Degradation Indicator: crack depth, x (not directly measurable)
Threshold of failure: x^{th}



Physical model of the degradation process



Paris-Erdogan model

$$\frac{dx}{dN} = e^{\omega} C \left(\beta \sqrt{x} \right)^{n} \longrightarrow \text{Discretization of the dynamics} \longrightarrow x_k = x_{k-1} + e^{\omega_{k-1}} C \left(\beta \sqrt{x_{k-1}} \right)^{n} \Delta N$$

- x = **hidden** degradation state (crack depth)
- ω = independent Gaussian **process noise**
- N = load cycle → ime k
- C, β and n = constants related to the material properties

Measurement equation

$$z_k = d\left[1 - \exp\left(\beta_0 + \beta_1 \ln\frac{x_k}{d - x_k} + \upsilon_k\right)\right]^{-1}$$

Logit model: non-destructive ultrasonic inspections

- z_k = degradation observation (vibration measurements)
- υ_k = independent non additive **measurement noise**
- β_0, β_1 = constants related to the material properties

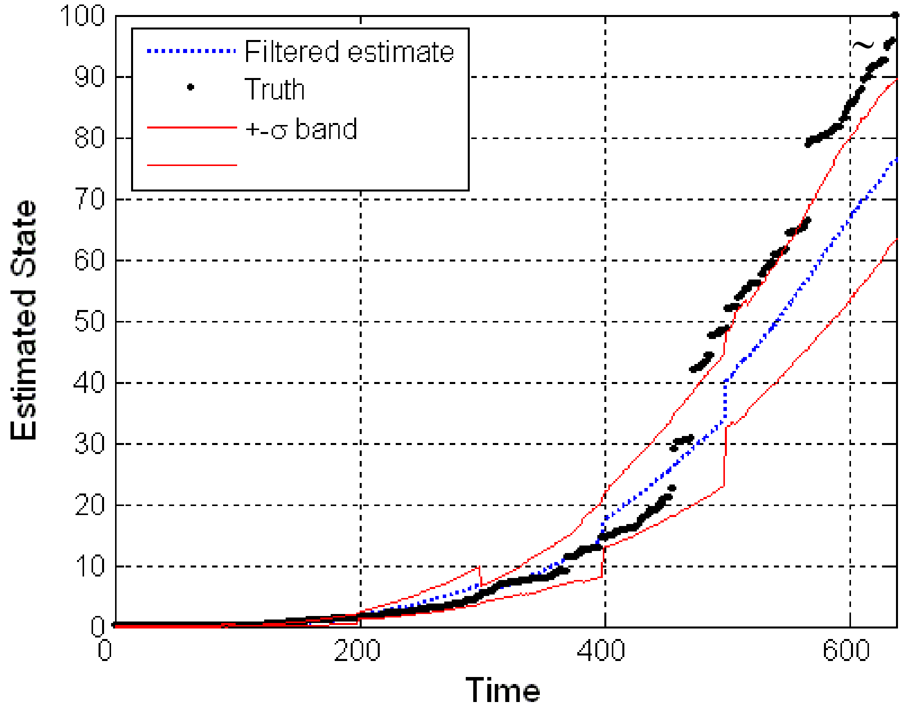
Objectives

- Degradation state (crack depth) estimate at the present time
- RUL prediction
- Maintenance planning

Crack growth evolution

- 5 measurements at: k_1 = 100; k_2 = 200; k_3 = 300; k_4 = 400; k_5 = 500
- 5000 particles

$$p\left(x_k \middle| z_{0:k}\right)$$

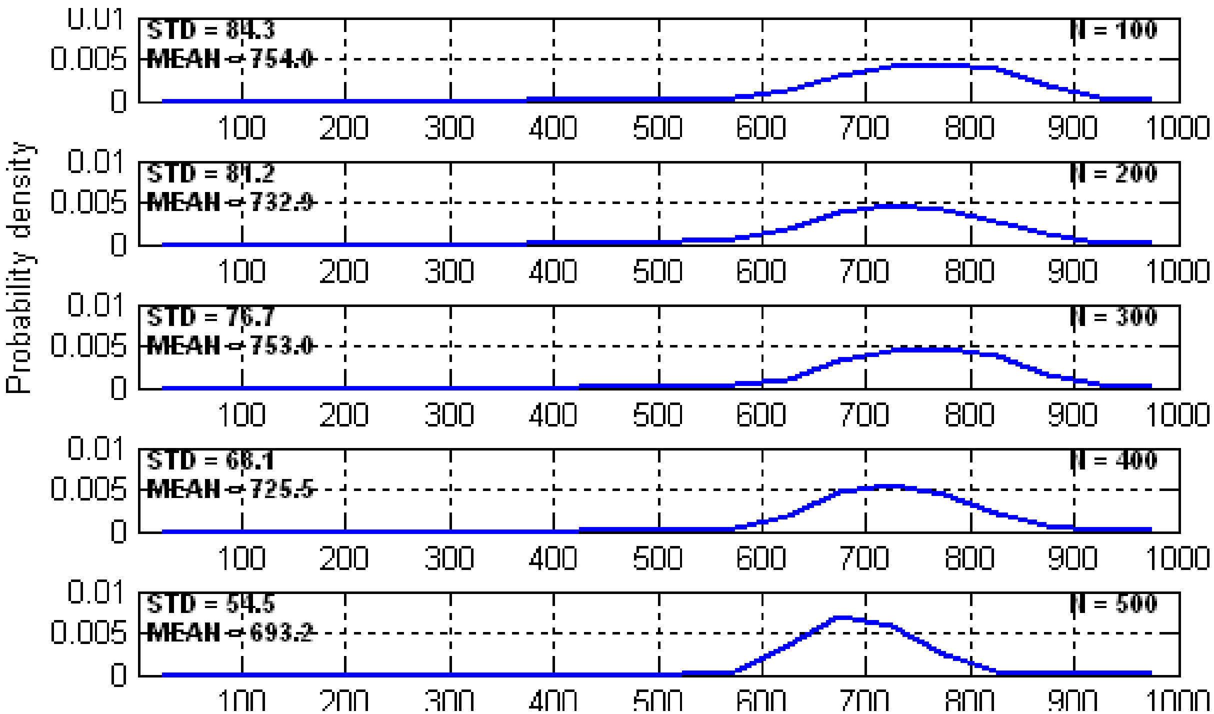


F. Cadini, E. Zio, D. Avram "*Monte Carlo-based filtering for fatigue crack growth estimation*", Probabilistic Engineering Mechanics, **24**, n. 3, pp. 367-373, 2009

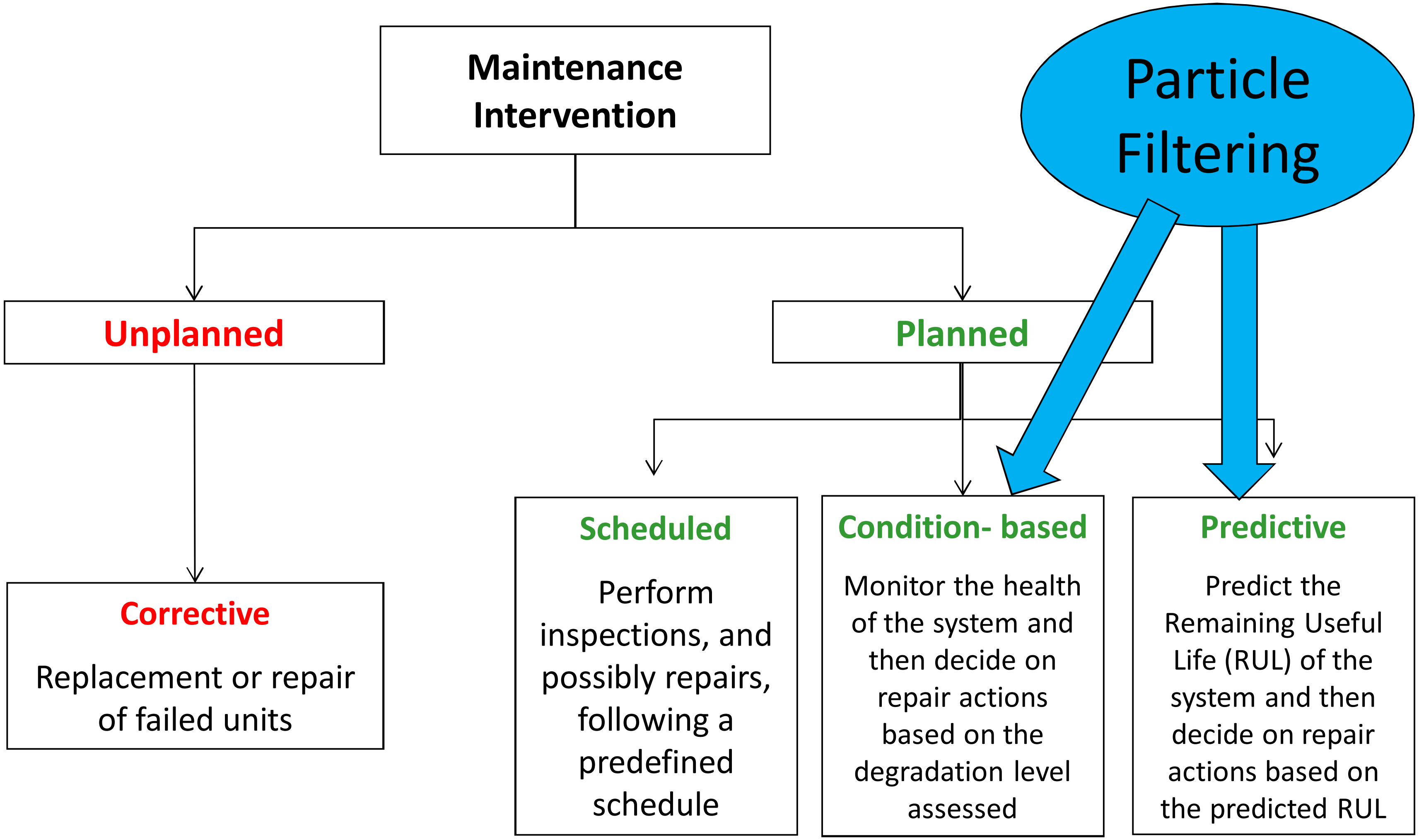
RUL estimate

Å 5 measurements at: k_1 = 100; k_2 = 200; k_3 = 300; k_4 = 400; k_5 = 500
Å 5000 particles
Å *True failure time is 631*

$p(t_f|z_{1:k})$



Maintenance: ultimate goal of PHM



Predictive maintenance

- A cost model of literature[*] is considered for the quantification of the costs driving the maintenance strategy

- Hypotheses:

  - **Inspection procedure**: periodic inspections are performed at given scheduled times. Results of the inspection are $z_{1:k}$.

  - **Maintenance actions**: either replacement upon failure (cost c_f) or preventive replacement (cost c_p)

  - **Decision-making policy**: at any future time a decision can be made on whether to replace the component or to further extend its life, albeit assuming the risk of a possible failure

[*] A.H. Christer, W. Wang, J.M. Sharp, A state space condition monitoring model for furnace erosion prediction and replacement, European Journal of Operational Research, Vol. 101, 1997, pp. 1-14

Predictive maintenance planning

- l is the remaining life duration until replacement
- Expected cost per unit time, $C(k,l)$ (evaluated at the present time k, assuming that the component will be replaced at time $k+l$)

$$C(k,l)= f(c_p, c_f, P(RUL<l))$$

Å $P(RUL<l)$ Particle filter!!

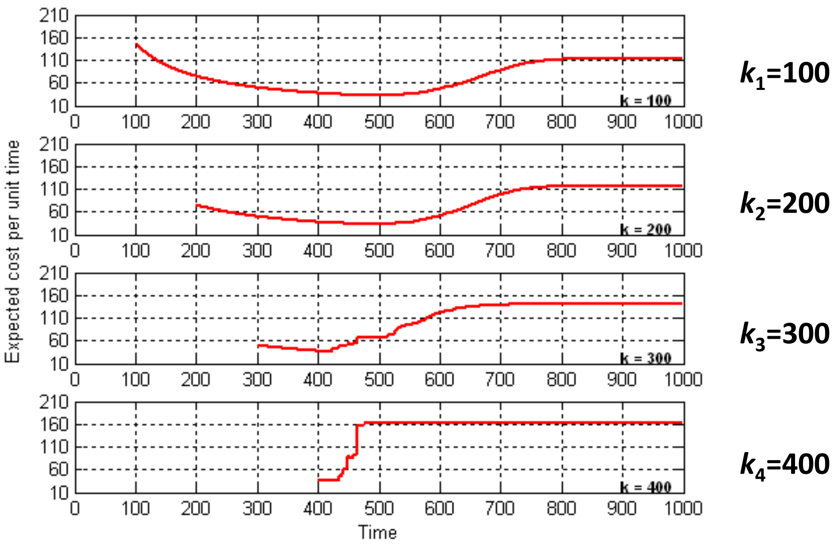
- Among all future time steps l, the best time to replacement l_{min} is the one which minimizes:

$$C(k,l)= f(c_p, c_f, P(RUL<l))$$

Predictive maintenance: results

- Measurements at time steps: $k_1 = 100$, $k_2 = 200$, $k_3 = 300$, $k_4 = 400$
- Number of particles: 5000
- TRUE FAILURE TIME = 452

Expected cost per unit time



Time step (k)	Minimum E[cost per unit time]	K_{min}
100	33	505
200	33	516
300	36	423
400	35	434

F. Cadini, E. Zio "*Model-based Monte Carlo state estimation for condition-based component replacement*", Reliability Engineering and System Safety, doi:10.1016/j.ress.2008.08.003, **94**, n. 3, pp. 752-758, 2009

Outline

- Prognostics
- Model-based prognostics
- Particle filtering for degradation state estimate
- Particle filtering for RUL estimate
- Application
  - Maintenance planning

Particle Filtering for Prognostics at PHME 2014

- Session 2c: «*Sequential Monte Carlo sampling for crack growth prediction providing for several uncertainties*» by: Matteo Corbetta, Claudio Sbarufatti, Andrea Manes, Marco Giglio
- Session 2c: «A Prognostic Approach Based on Particle Filtering and optimized Tuning Kernel Smoothing» Yang Hu, Piero Baraldi, Francesco Di Maio, Enrico Zio
- Session 5b: «A particle Filtering-based Approach for the prediction of the Remaining Useful Life of an Aluminium Electrolytic Capacitor» Marco Rigamonti, Piero Baraldi, Enrico Zio, Daniel Astigarraga, Ainhoa Galarza
- Session 8b: «*A Model-Based Prognostics Framework to Predict Fatigue Damage Evolution and Reliability in Composites*» by Juan Chiach, Manuel Chiach, Abhinav Saxena, Guillermo Rus and Kai Goebel

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- Yang Hu
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