UTC Aerospace Systems

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# Theoretical Aspects of Prognostics

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# Introduction & Motivation

## Failure Prognostics & ToF PMF Computation

- Key Constituent Elements:
  - Knowledge about current system condition (Bayesian processors)
  - Characterization of future operating profiles (Stochastic processes)
  - System degradation model
    - ✓ Model structure
    - ✓ Model parameters
  - Module for uncertainty characterization in future state transitions
    - ❑ Monte Carlo, rare event simulation algorithms
    - ❑ PF-based, Sigma-points-based, NN-based prognostic algorithms

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- Dynamic Model for Feature Growth in Time:

$$\begin{cases} x_1(t+1) = x_1(t) + x_2(t) \cdot F(x_1(t), t, U) + \omega_1(t) \\ x_2(t+1) = x_2(t) + \omega_2(t) \end{cases}$$

  - $x_1(t)$ is a state representing the fault dimension under analysis
  - $x_2(t)$ is a state associated with an unknown model parameter
  - $U$ are external inputs to the system (load profile, etc.)
  - $F(x(t), t, U)$ is a general time-varying nonlinear function
  - $\omega_1(t)$ and $\omega_2(t)$ are white noises (non necessarily Gaussian)

- Predicted State Density:

$$\tilde{p}(x_{t+p} \mid y_{1:t}) = \int \tilde{p}(x_t \mid y_{1:t}) \prod_{j=t+1}^{t+p} p(x_j \mid x_{j-1}) dx_{t:t+p-1}$$

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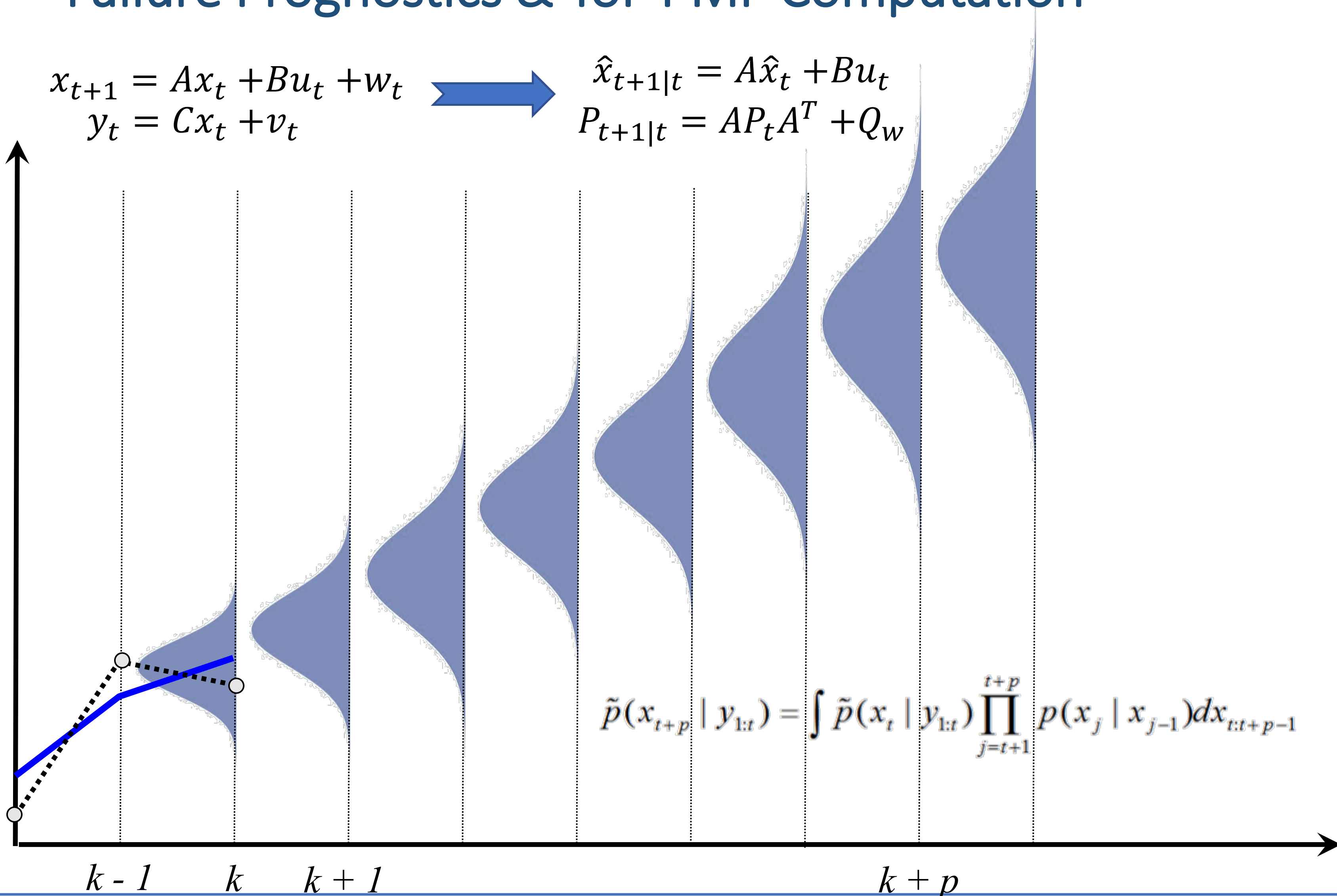
## Failure Prognostics & ToF PMF Computation

$$x_{t+1} = Ax_t + Bu_t + w_t$$
$$y_t = Cx_t + v_t$$

$$\hat{x}_{t+1|t} = A\hat{x}_t + Bu_t$$
$$P_{t+1|t} = AP_tA^T + Q_w$$



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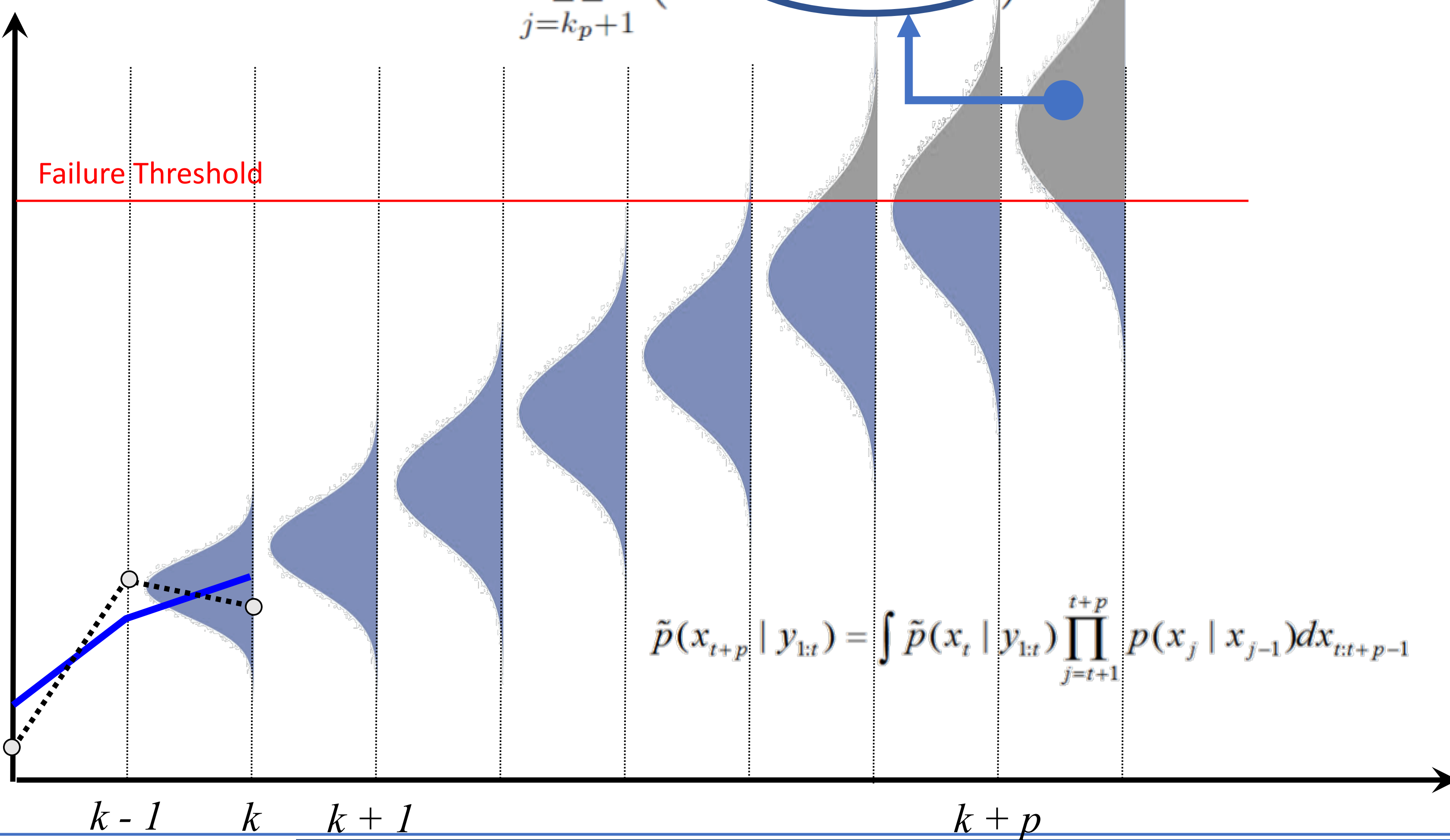
## Failure Prognostics & ToF PMF Computation

$$\mathcal{P}(\mathcal{F}_k) = \mathcal{P}(\mathcal{F}_k|\mathcal{H}_{k_p:k-1}) \prod_{j=k_p+1}^{k-1} \left(1 - \mathcal{P}(\mathcal{F}_j|\mathcal{H}_{k_p:j-1})\right)$$




$$\tilde{p}(x_{t+p} \mid y_{1:t}) = \int \tilde{p}(x_t \mid y_{1:t}) \prod_{j=t+1}^{t+p} p(x_j \mid x_{j-1}) dx_{t:t+p-1}$$

# Introduction & Motivation

## Failure Prognostics & ToF PMF Computation

We denote $\mathcal{H}_k$ as the event of *being in a faulty, although operative*, condition at time $k$, whereas $\mathcal{F}_k$ denotes the event of *undergoing a catastrophic failure* at time $k$.

We can define a probability space $(\Omega, \mathcal{B}, \mathcal{P})$, where

- $\Omega = \{\left(\bigcup_{j=k_p}^{k-1} \mathcal{H}_j\right) \bigcup \mathcal{F}_k |\ k \in \mathbb{N},\ 0 < k_p < k\}$ is the sample space that determines all possible sequences where the system remains operative until the time instant "$k$", when it actually undergoes a catastrophic failure,
- $\mathcal{B} = \sigma(\Omega)$ is the $\sigma$-algebra generated by $\Omega$,
- $\mathcal{P}$ is a function that assigns a probability measure to every event in the $\sigma$-algebra $\mathcal{B}$.

# Introduction & Motivation

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Denoting $\mathcal{H}_{k_p:k} = \bigcup_{j=k_p}^{k} \mathcal{H}_j$, and according to the definition of conditional probability, it follows that:

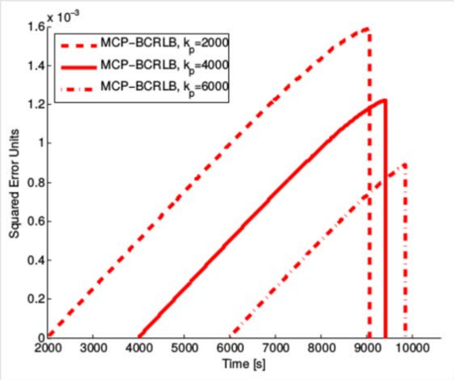
$$\mathcal{P}(\mathcal{F}_k) = \frac{\mathcal{P}(\mathcal{F}_k, \mathcal{H}_{k_p:k-1})}{\mathcal{P}(\mathcal{H}_{k_p:k-1} | \mathcal{F}_k)}, \quad \forall k > k_p \tag{1}$$

It is assumed that the system can only experiment one catastrophic failure, then $\mathcal{P}(\mathcal{H}_{k_p:k-1} | \mathcal{F}_k) = 1$. Hence,

$$\mathcal{P}(\mathcal{F}_k) = \mathcal{P}(\mathcal{F}_k, \mathcal{H}_{k_p:k-1}) = \mathcal{P}(\mathcal{F}_k | \mathcal{H}_{k_p:k-1}) \mathcal{P}(\mathcal{H}_{k_p:k-1}), \quad \forall k > k_p \tag{2}$$

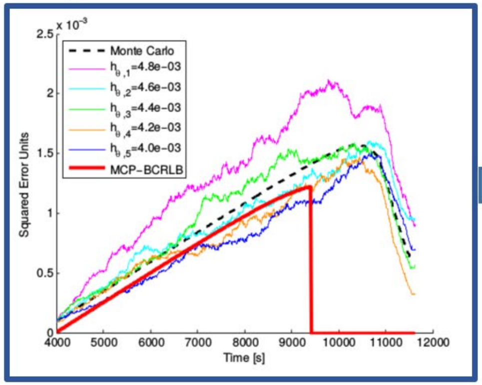
Compute (recursively) predictive Bayesian Cramér-Rao Lower Bounds for state MSE

$$x_k = f(x_{k-1}, \omega_{k-1})$$



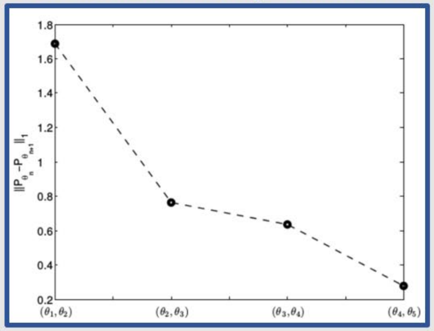

Prognostic algorithm with hyper-parameter vector $\theta = \left[\theta_A^T \quad \theta_B^T\right]^T$

- Fix $\theta_A$ using efficiency criteria and generate hyper-parameters candidates for $\theta_B$
- Compare predictive MSE and MCP-BCRLBs




Relax efficiency constraints (change $\theta_A$)

Evaluate performance of prognostic algorithm in terms of the estimated Time-of-Failure PMF (effectiveness)




Efficiency and efficacy constraints satisfied

Design procedure concluded